Digital Signal Processing Summer Seminar
Institute of Information Technology - Vietnam Education Foundation

FUNDAMENTALS OF SIGNAL PROCESSING

Problem Set 3 – Digital Filtering

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Problem 1

A linear time-invariant filter is used to process sampled data (with sampling interval T), and is described by the the difference equation:

$$y_n = -0.5y_{n-1} + 0.5u_n + u_{n-1}$$

(a) Determine the transfer function $H(z)$ for the system. Express $H(z)$ as a ratio of polynomials in z^{-1}, and also as a ratio of polynomials in z.
(b) Plot the poles and zeros of $H(z)$ in the z-plane.
(c) Is this a stable system?
(d) From $H(z)$, and the system frequency response function, and show that this is an "all-pass" system, that is $\left|H^*(j\omega)\right| = 1$ for all $|\omega| \le \pi/T$. Determine the system phase response at frequencies $\omega = 0$ and $\omega = \pi/T$.

Problem 2

Consider a filter with the following impulse response, where $b[n]$ is a real-valued sequence of length M:

$$h[n] = \begin{cases} b[n] & 0 \le n \le M-1 \\ -b[2M-1-n] & M \le n \le 2M-1 \\ 0 & otherwise \end{cases}$$

Consider whether or not $H(e^{j\omega})$ can be writtent as

$$H(e^{j\omega}) = A(e^{j\omega})e^{j(\alpha\omega+\theta)}$$

For real-valued $A(e^{j\omega})$, and real constants α, and θ. If $H(e^{j\omega})$ can be written in the above, find $A(e^{j\omega})$, α, and θ. If not say why not.

Problem 3

(a) Design a length-7 differentiator filter, which has desired generalized amplitude response $H_d(\omega) = j\omega \quad , \quad -\pi < \omega < \pi$
using the window design method using a Hamming window. (Be sure to incorporate linear phase in your design.) Is the resulting design desirable? Do you expect substantially different results using frequency sampled design?
(b) Design a length-8 differentiator filter using the frequency sampled design method with equally spaced samples $\omega_k = 2\pi k/8$, using whichever symmetry seems most appropriate.

Problem 4

Design a length-3, symmetric equiripple high-pass filter with the following $A_d(\omega)$ with a stopband edge of $\pi/3$ and a passband edge of $\pi/2$, and with uniform weighting of the stopband and passband.
What are the filter coeffcients and the ripple amplitude δ?

$$A_d(\omega) = 1, \quad \frac{\pi}{2} \le \omega \le \pi$$
$$= 0, \quad otherwise.$$

Hint: You do not need Parks-McCellan algorithm to solve this problem.

Problem 5
Experiments with frequency sampled design: For all of the following parts, do a length-3 frequency sampled design, with desired magnitude response of

$$|H_d(\omega)| = \begin{cases} 0 : |\omega| \le \frac{\pi}{2} \\ 1 : \frac{\pi}{2} < |\omega| \le \pi. \end{cases}$$

Assume linear phase and symmetry unless ottherwise directed.

(a) Design a filter with equally spaced frequency samples, $\omega_k = \frac{2\pi k}{3}$. Sketch the frequency response.
(b) Design a filter with equally spaced frequency samples, $\omega_k = \frac{2\pi(k+\frac{1}{2})}{3}$. Sketch the frequency response.
(c) The above designs overshoot at $\omega = 0, \pi$. Do a frequency sampled design for frequency samples at $\omega = 0, \pi$. Sketch the frequency response. Has the overshoot been reduced?
(d) Unfortunately , the transition band is very broad. Try to correct fot that by choosing frequency sample locations at $\omega = \frac{\pi}{4}, \frac{3\pi}{4}$. Sketch the frequency response.
(e) Try to force a very narrow transition band by choosing sample locations $\omega = 0.45\pi, 0.55\pi$. Sketch the frequency response. Are the results desirable?

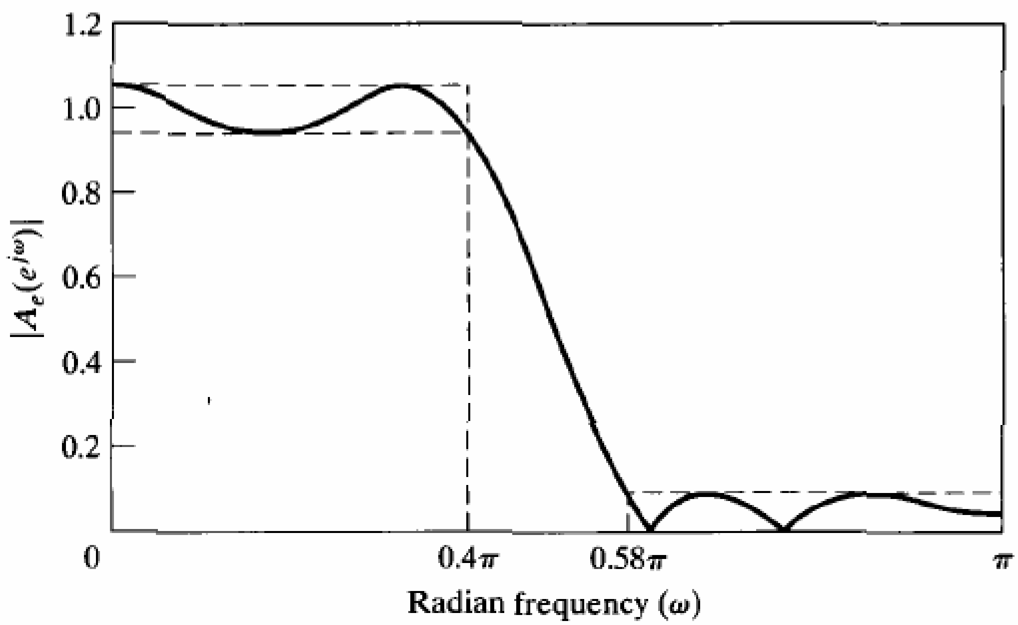
Problem 6
Suppose that we have to design a length-N FIR Filter $\{h[n]\}_{n=0}^{N-1}$ (whose coefficients can be complex-valued) so that its DTFT samples at $\omega_k = \frac{\pi(2k+1)}{N}$, $k = 0,1,...,N-1$, are equal to specified values:
$H(\omega_k) = b_k, \quad k=0,1,...,N-1$

(a) Set up this filter design problem as a matrix equation $Ax = b$ anf specified all the entries in A, B, and x.

(b) Instead of inverting the above matrix equation, exploit the equally spaced prpoerty of $\{\omega_k\}_{k=0}^{N-1}$ to derive an $O(N \log N)$ algorithm for this filter design problem using the inverse FFT.

Problem 7

An optimal equiripple FIR linear-phase filter was designed by Parks-McClellan algorithm. The magnitude of this frequency response is show in the below figure. The maximum approximation error in the passband is $\delta_1 = 0.0531$, and the maximum approximation error in the stopband is $\delta_2 = 0.085$. The passband and stopband cutof f frequencies are $\omega_p = 0.4\pi$ and $\omega_s = 0.58\pi$.



(a) What type (I, II, III or IV) of the linear-phase system is this? Explain how you can tell.

(b) What was the error-weight function $W(\omega)$ that was used in optimization?

(c) Sketch the weighted approximation error; i.e., sketch

$$E(\omega) \approx W(\omega)[H_d(e^{j\omega}) - A_e(e^{j\omega})].$$

Where $H_d(e^{j\omega})$ is the frequency response of the corresponding ideal low pass filter

(d) What is the length of the impulse response of the system?