

Digital Signal Processing Summer Seminar
 Institute of Information Technology - Vietnam Education Foundation
FUNDAMENTALS OF SIGNAL PROCESSING
Problem Set 1 - Foundations
 July 2006

Problem 1

A linear time-invariant system has frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega 3}, & |\omega| < \frac{2\pi}{16} \left(\frac{3}{2}\right) \\ 0, & \frac{2\pi}{16} \left(\frac{3}{2}\right) \leq |\omega| \leq \pi \end{cases}$$

The input to the system is a periodic unit-impulse train with period $N = 16$; i.e.,

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k].$$

Find the output of the system.

Problem 2

Consider a stable LTI system frequency $H(\omega)$. Let $y[n]$ be the output of the system given the following input

$$x[n] = \begin{cases} e^{j\omega_0 n} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Show that $y[n] \rightarrow H(\omega_0)e^{j\omega_0 n}$ as $n \rightarrow \infty$

Problem 3

Consider the signal $x[n]$ shown in Figure 1. Note that $x[n]$ is zero for all samples which are shown in the figure.

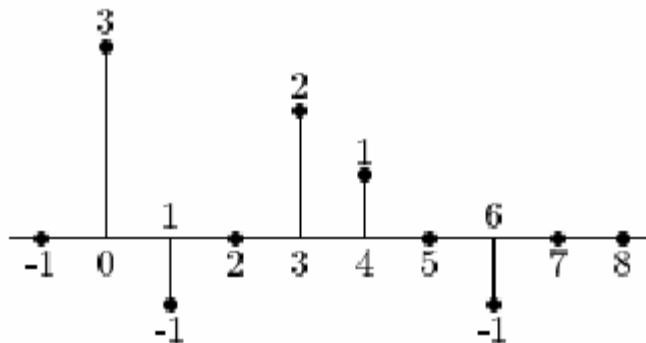


Figure 1, $x[n]$

(a) Let $X(e^{j\omega})$ be the DTFT of the infinite-length signal $x[n]$, Define

$$R[k] = X(e^{j\omega}) \Big|_{\omega = \frac{2\pi k}{4} + \frac{2\pi}{8}}, \quad 0 \leq k \leq 3.$$

Determine the signal $r[n]$ which is the four-point inverse DFT of $R[k]$.

- (b) Let $X[k]$ be the eight-point DFT of $x[n]$, $0 \leq n \leq 7$, i.e. $X[k] = \text{DFT}(\{3, -1, 0, 2, 1, 0, -1, 0\})$ and let $H[k]$ be the eight-point DFT of the impulse response $h[n]$, $0 \leq n \leq 7$, shown in Figure 2. Define $Y[k] = H[k]X[k]$ for $0 \leq k \leq 7$. determine $y[n]$, the eight-point inverse DFT of $Y[k]$.

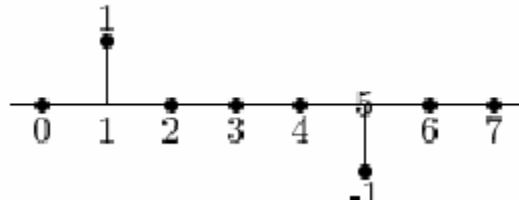


Figure 2, $h[n]$

- (c) Again, let $X[k]$ be the eight-point DFT of $x[n]$, $0 \leq n \leq 7$, as in part (b). Define

$$W[k] = \begin{cases} X[k], & 0 \leq k \leq 7, \\ X[k-8], & 8 \leq k \leq 15. \end{cases}$$

Determine and sketch $w[n]$, the 16-point inverse DFT of $W[k]$.

Problem 4

- (a) Suppose $x[n] = 0$ $n < 0, n > (N-1)$ is an N -point sequence having at least one nonzero sample. Is it possible for such a sequence to have a DTFT

$$X(e^{j2\pi k/M}) = 0 \quad k = 0, 1, \dots, M-1$$

Where M is an integer greater than or equal to N ? If your answer is yes, construct an example. If your answer is no, explain your reason.

- (b) Suppose $M < N$. Repeat part (a)

Problem 5

Suppose you have an efficient computer program (or hardware device) for computing the FFT of length N . Describe how would you use this program (or device) to compute N equally spaced samples between 0 and 2π , $\omega_k = \frac{2\pi k}{N}$ ($0 \leq k < N$), of the DTFT of an input signal of length M , $\{x[n]\}_{n=0}^{M-1}$. Be sure to consider both case $M \leq N$ and $M > N$.

Problem 6

A length- N signal $\{x[n]\}_{n=0}^{N-1}$ is zero-padded at the end to length $2N$ signal $\{y[n]\}_{n=0}^{2N-1}$ and the corresponding DFT $\{Y[k]\}_{k=0}^{2N-1}$ is computed. The length- N inverse DFT of the

sequence of odd-index components $\{Y[2k+1]\}_{k=0}^{N-1}$ is then computed. Describe the final result directly in terms of $\{x[n]\}_{n=0}^{N-1}$