Digital Signal Processing Summer Seminar
Institute of Information Technology - Vietnam Education Foundation

FUNDAMENTALS OF SIGNAL PROCESSING

Problem Set 1 - Foundations

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Problem 1

A linear time-invariant system has frequency response

$$H\left(e^{j\omega}\right)=\begin{cases} e^{-j\omega 3}, & |\omega|<\frac{2\pi}{16}\left(\frac{3}{2}\right) \\ 0, & \frac{2\pi}{16}\left(\frac{3}{2}\right)\le|\omega|\le\pi \end{cases}$$

The input to the system is a periodic unit-impulse train with period $N = 16$; i.e.,

$$x[n]=\sum_{k=-\infty}^{\infty}\delta[n+16k].$$

Find the output of the system.

Problem 2

Consider & stable LTI system frequence $H(\omega)$. Let $y[n]$ be the output of the system given the following input

$$x[n]=\begin{cases} e^{j\omega_0 n} & n\ge 0 \\ 0 & n<0 \end{cases}$$

Show that $y[n]\to H(\omega_0)e^{j\omega_0 n}$ as $n\to\infty$

Problem 3

Consider the signal $x[n]$ shown in Figure 1. Note that x[n] is zero for all samples which are shown in the figure.

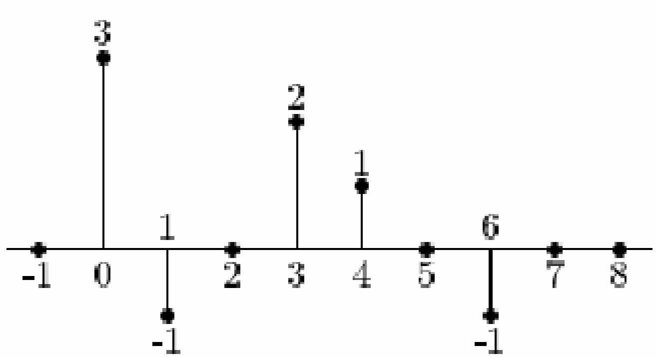

Figure 1, x[n]

(a) Let $X(e^{j\omega})$ be the DTFT of the infinite-length signal $x[n]$, Define

$$R[k] = X(e^{j\omega})\big|_{\omega=\frac{2\pi k}{4}+\frac{2\pi}{8}}, \quad 0 \le k \le 3.$$

Determine the signal $r[n]$ which is the four-point inverse DFT of $R[k]$.

(b) Let $X[k]$ be the eight-point DFT of $x[n]$, $0 \le n \le 7$, i.e. $X[k] = DFT$ ({3,-1,0,2,1,0,-1,0}) and let $H[k]$ be the eight-point DFT of the impulse response $h[n]$, $0 \le n \le 7$, shown in Figure 2. Define $Y[k] = H[k]X[k]$ for $0 \le k \le 7$. determine $y[n]$, the eight-point inverse DFT of $Y[k]$.

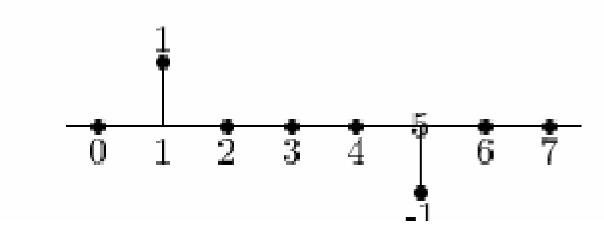

Figure 2, h[n]

(c) Again, let $X[k]$ be the eight-point DFT of $x[n]$, $0 \le n \le 7$, as in part (b). Define

$$W[k] = \begin{cases} X[k], & 0 \le k \le 7, \\ X[k-8], & 8 \le k \le 15. \end{cases}$$

Determine and sketch $w[n]$, the 16-point inverse DFT of $W[k]$.

Problem 4

(a) Suppose $x[n] = 0 \qquad n < 0,\ n > (N-1)$ is an N-point sequence having at least one nonzero sample. Is it possible for such a sequence to have a DTFT

$$X(e^{j2\pi k/M}) = 0 \qquad k = 0,1,\ldots,M-1$$

Where M is an integer greater than or equal to N? If your answer is yes, construct an example. If your answer is no, explain your reason.

(b) Suppose M<N. Repeat part (a)

Problem 5

Suppose you have an efficient computer program (or hardware device) for computing the FFT of length N. Describe how would you use this program (or device) to compute N equally spaced samples between 0 and 2π, $\omega_k = \dfrac{2\pi k}{N}\left(0 \le k < N\right)$, of the DTFT of an input signal of length M, $\left\{x[n]\right\}_{n=0}^{M-1}$. Be sure to consider both case $M \le N$ and $M > N$.

Problem 6

A length-N signal $\left\{x[n]\right\}_{n=0}^{N-1}$ is zero-padded at the endn to length $2N$ signal $\left\{y[n]\right\}_{n=0}^{2N-1}$ and the corresponding DFT $\left\{Y[k]\right\}_{k=0}^{2N-1}$ is computed. The length-N inverse DFT of the sequence of odd-index components $\left\{Y\left[2k+1\right]\right\}_{k=0}^{N-1}$ is then computed. Describe the final result directly in terms of $\left\{x\left[n\right]\right\}_{n=0}^{N-1}$