# Image Coding, JPEG, and JPEG2000

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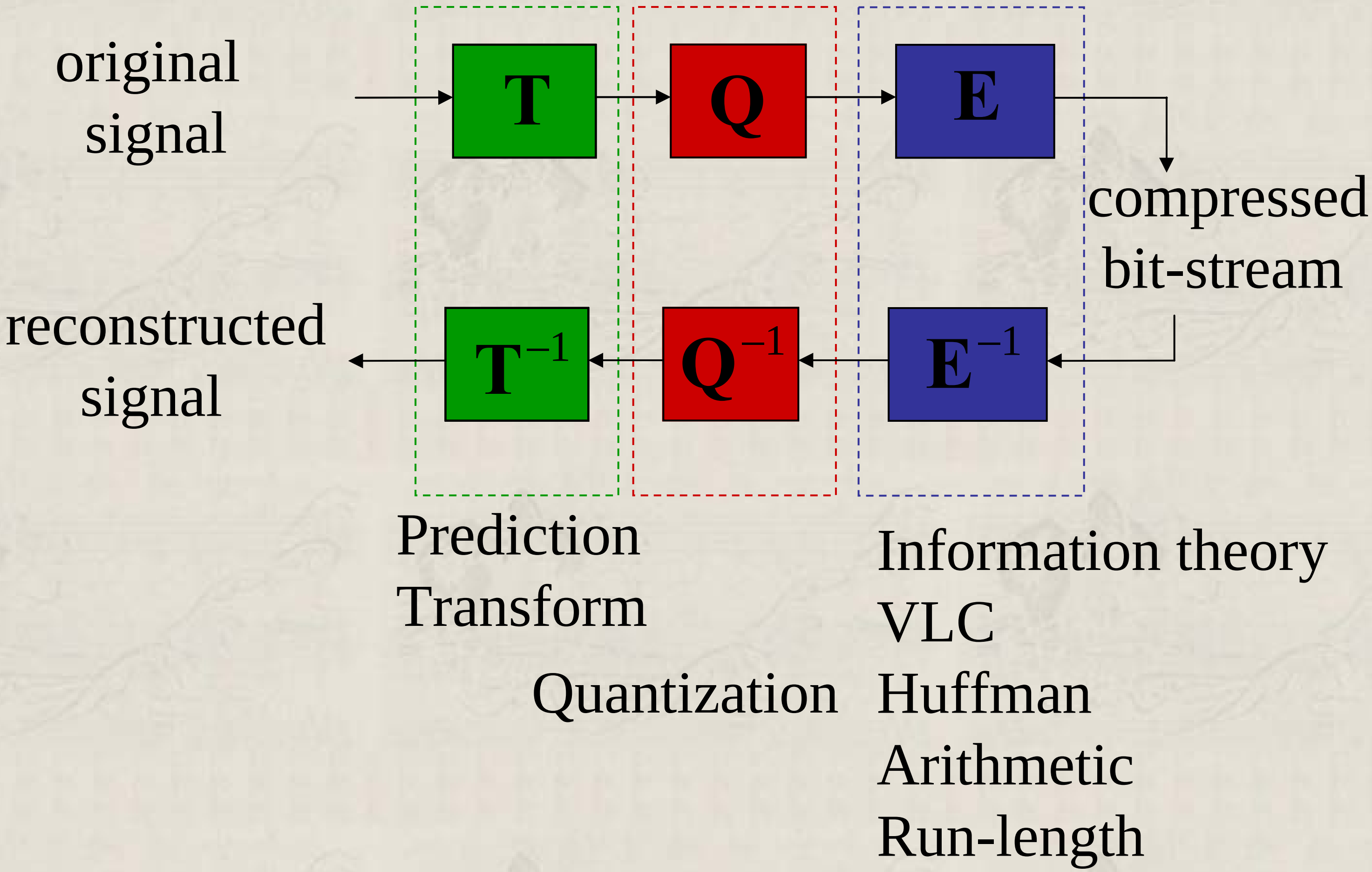
# Outline

- Prediction
  - Open-loop differential pulse-code modulation (DPCM)
  - Closed-loop DPCM
  - Optimal linear prediction
- Transformation
  - Transform fundamentals
  - Karhunen-Loeve Transform (KLT): optimal linear transform
  - Discrete Cosine Transform (DCT)
- Putting everything together
  - JPEG:  DCT – Quantization – Run-length – Huffman
- Latest development
  - JPEG2000 and scalable, progressive coding

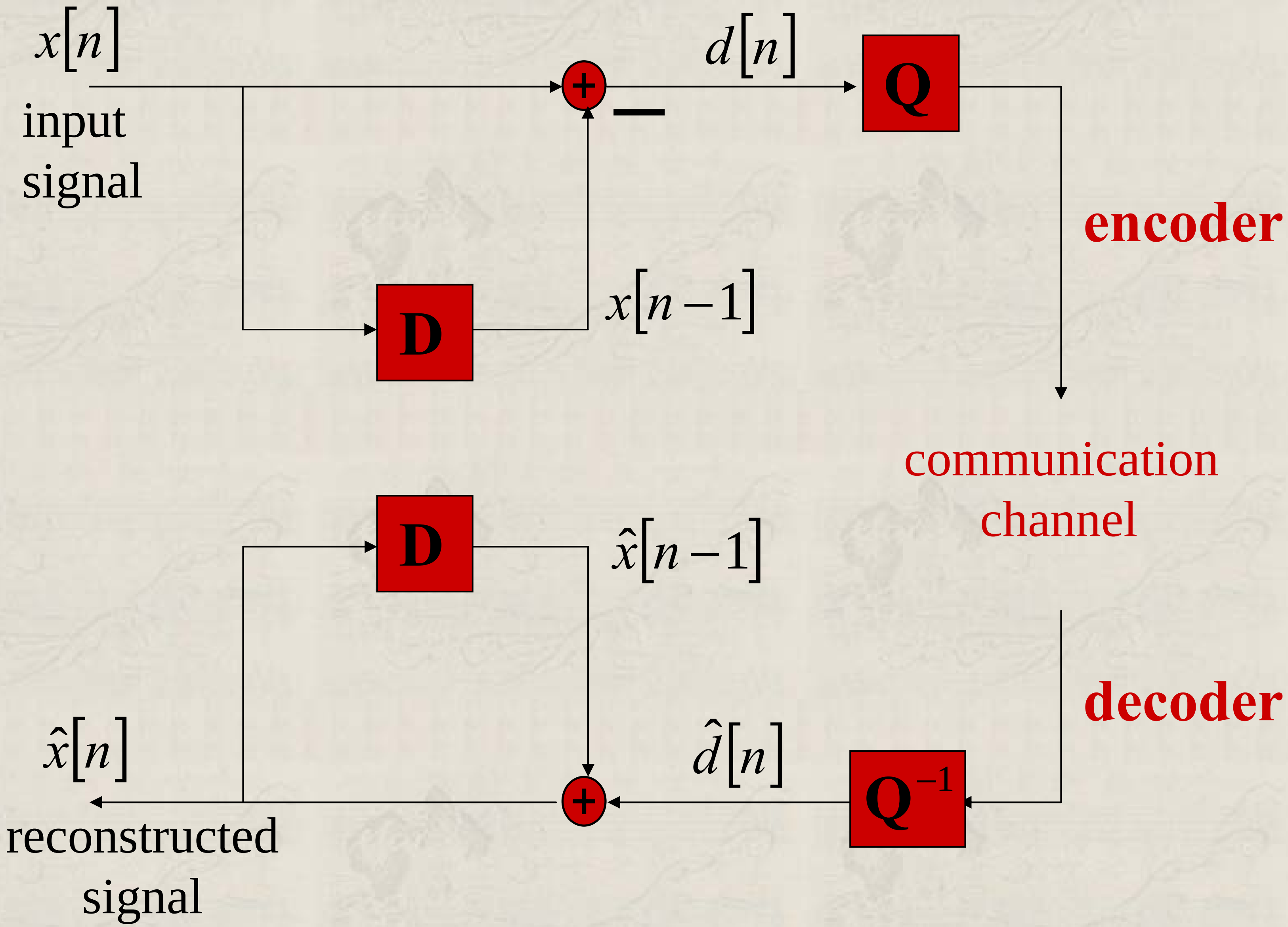
# Reminder


original
signal
T
Q
E
compressed
bit-stream
reconstructed
signal
$T^{-1}$
$Q^{-1}$
$E^{-1}$
Prediction
Transform
Quantization
Information theory
VLC
Huffman
Arithmetic
Run-length

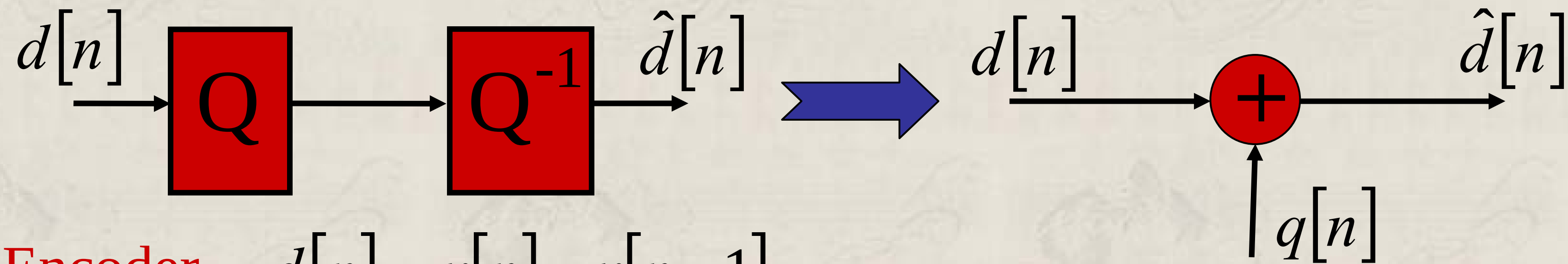
# Predictive Coding

- We have only dealt with memory-less model so far
  - Each symbol/sample is quantized and/or coded without much knowledge on previous ones
- There is usually a strong correlation between neighboring symbols/samples in multimedia signals
- Simplest prediction scheme: take the difference!
- If the difference between two adjacent symbols/samples is quantized and encoded instead, we can achieve the same level of compression performance using fewer bits – the range of the differences should be a lot smaller.

# Open-Loop DPCM



# Open-Loop DPCM: Analysis




Encoder $d[n] = x[n] - x[n-1]$

Decoder $\hat{x}[n] = \hat{d}[n] + \hat{x}[n-1]$

$$\hat{d}[n] = d[n] + q[n]$$

$$\hat{d}[0] = d[0] + q[0] \qquad \hat{x}[0] = \hat{d}[0] = d[0] + q[0] = x[0] + q[0]$$

$$\hat{d}[1] = d[1] + q[1] \qquad \hat{x}[1] = \hat{d}[1] + \hat{x}[0] = d[1] + q[1] + x[0] + q[0]$$

$$= x[1] - x[0] + q[1] + x[0] + q[0]$$

$$= x[1] + q[0] + q[1]$$

Quantization Error Accumulation

$$\hat{x}[n] = x[n] + \sum_{i=0}^{n} q[i] \quad \Rightarrow \quad |e[n]| = |\hat{x}[n] - x[n]| = \left|\sum_{i=0}^{n} q[i]\right|$$

# Open-Loop DPCM: Analysis

- There seems to be a **model mismatch**
  - **Encoder:** $d[n] = x[n] - x[n-1]$
  - **Decoder:** $\hat{x}[n] = \hat{d}[n] + \hat{x}[n-1]$

$$\Rightarrow \hat{d}[n] = \hat{x}[n] - \hat{x}[n-1]$$

- How about using the reconstructed sample for the difference?

$$d[n] = x[n] - \hat{x}[n-1]$$

# Closed-Loop DPCM: Analysis

Modified prediction scheme $$d[n] = x[n] - \hat{x}[n-1]$$

Decoder remains the same $$\hat{x}[n] = \underbrace{\hat{d}[n]}_{d[n]+q[n]} + \hat{x}[n-1]$$

$$\hat{d}[0] = d[0] + q[0] \qquad \hat{x}[0] = \hat{d}[0] = d[0] + q[0] = x[0] + q[0]$$

$$\hat{d}[1] = d[1] + q[1] \qquad \hat{x}[1] = \hat{d}[1] + \hat{x}[0] = d[1] + q[1] + x[0] + q[0]$$

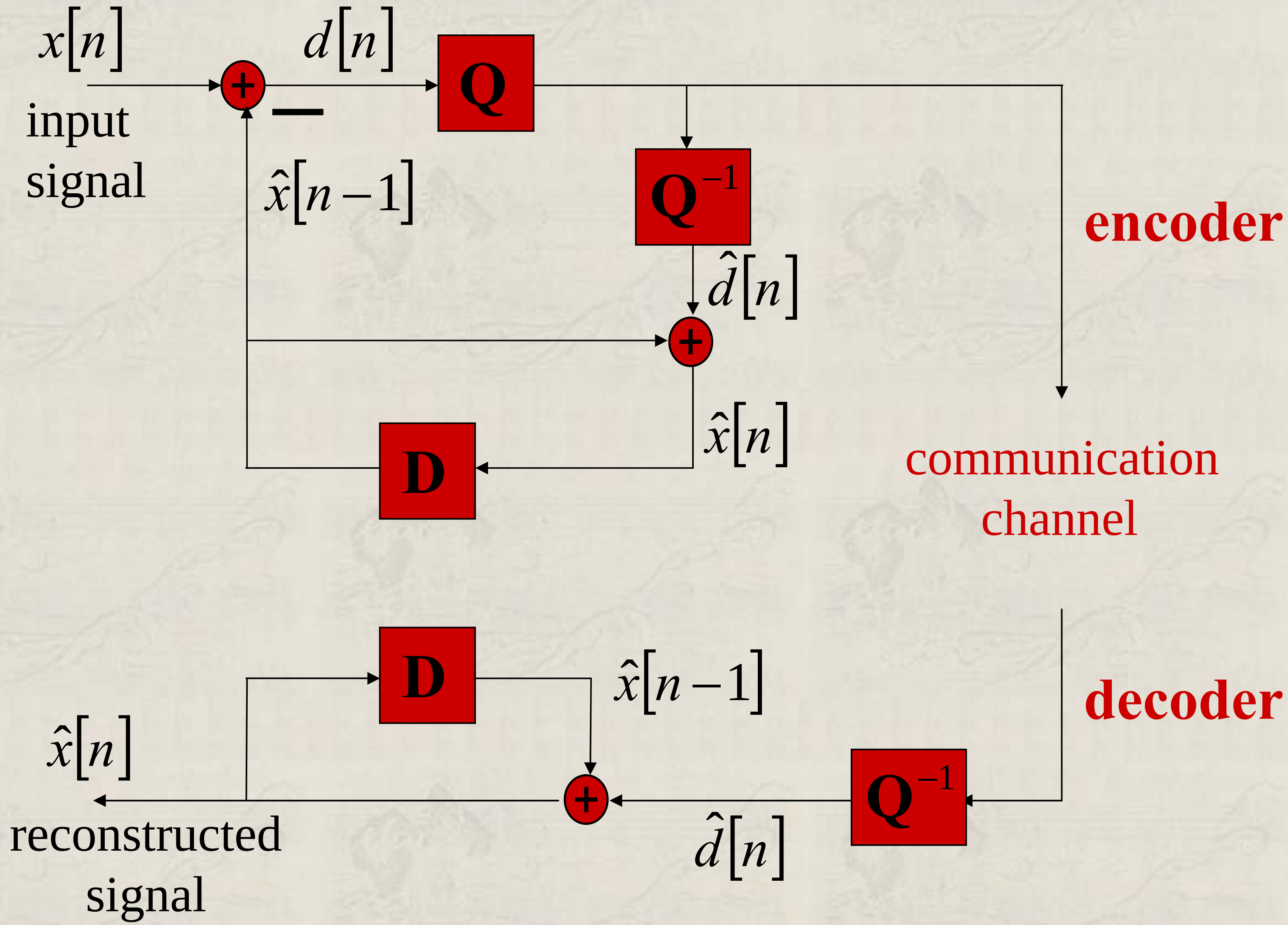
$$= x[1] - \boxed{\hat{x}[0]} + q[1] + x[0] + q[0]$$

$$= x[1] + q[1]$$

$$\boxed{\hat{x}[n] = x[n] + q[n]}$$

No error accumulation!

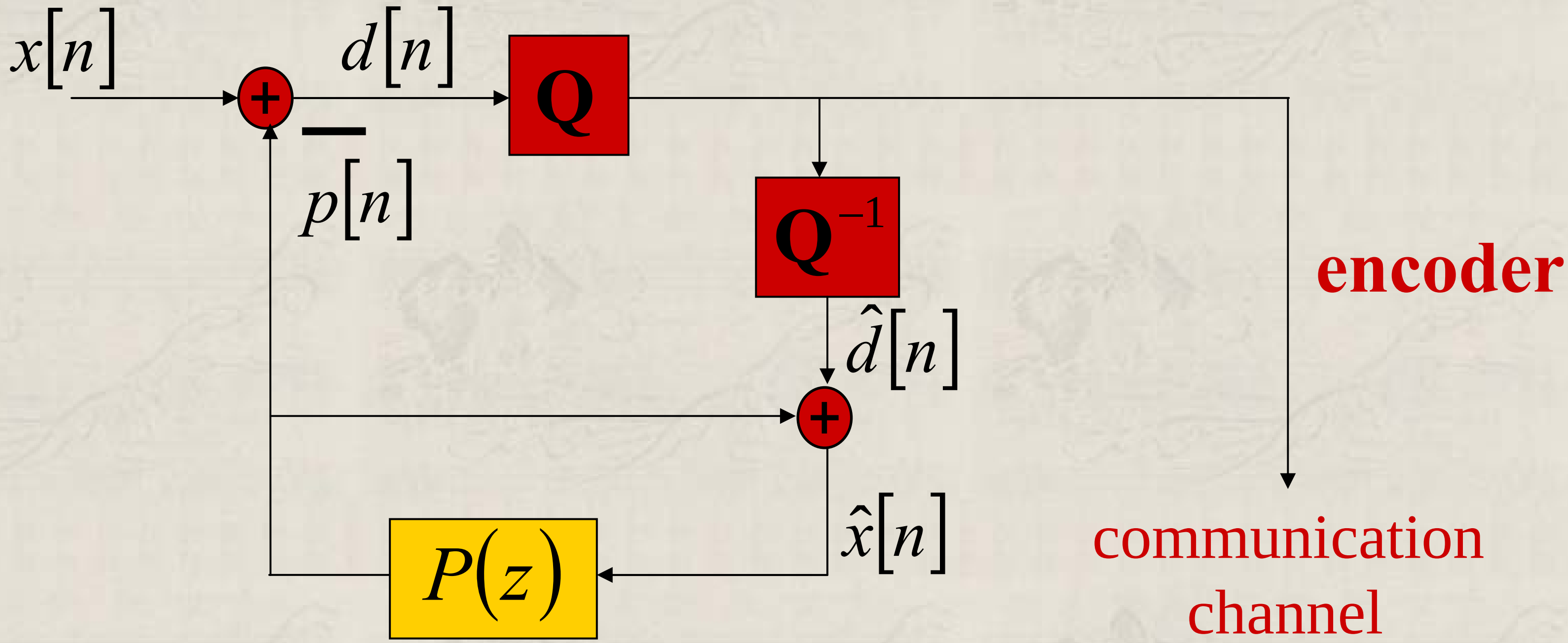
# Closed-Loop DPCM



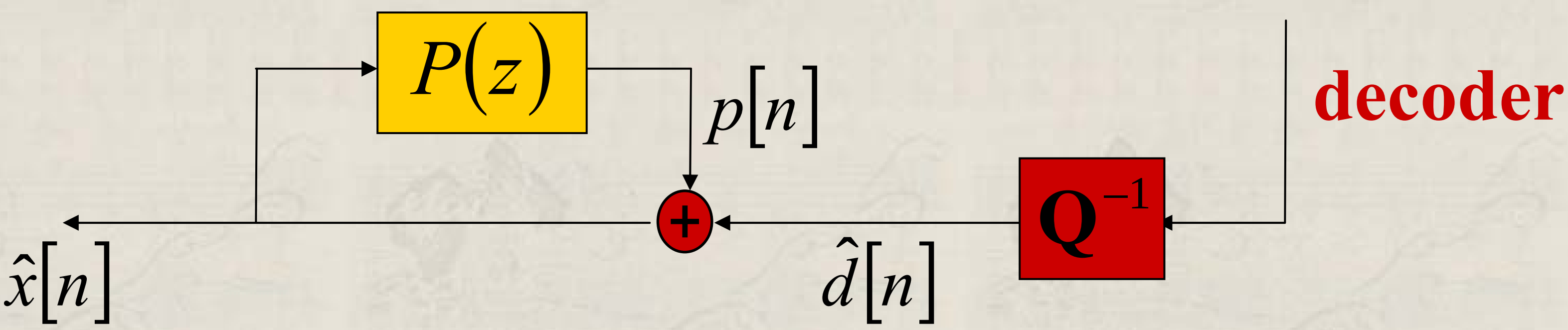
# Closed-Loop DPCM: Observations

- Quantization error does not accumulate
- Minor modification in prediction scheme leads to major encoder modification
  - Encoder now has decoder embedded inside
- Closed-loop & open-loop DPCM has the same decoder
- DPCM predicts current sample from last reconstructed one
- Generalization?
  - Replace the simple delay operator by more complicated & more sophisticated predictor $P(z)$

# Linear Prediction




$$p[n] = \{\hat{x}[n] \mid \hat{x}[n-1], \hat{x}[n-2], \ldots, \hat{x}[n-N]\} \xrightarrow{\textit{linear}} \sum_{i=1}^{N} a_i \hat{x}[n-i]$$



# Optimal Linear Prediction

- Problem

$$\text{Find } \{a_i\} \text{ s.t. } D = \sigma_d^2 = E\left[\left(x[n] - \sum_{i=1}^{N} a_i \hat{x}[n-i]\right)^2\right] \text{ is minimized}$$

- Assumptions
  - Signal is WSS $\quad R_{xx}(k) = E[x[n]x[n+k]]$
  - High bit rates, i.e., fine quantization

$$p[n] = \sum_{i=1}^{N} a_i \hat{x}[n-i] \approx \sum_{i=1}^{N} a_i x[n-i]$$

- Approach $\quad \dfrac{\delta}{\delta a_i} D = 0$

# Optimal Linear Prediction

$$\frac{\delta}{\delta a_i} D = \frac{\delta}{\delta a_i} E\left[\left(x[n] - \sum_{i=1}^{N} a_i \hat{x}[n-i]\right)^2\right] = 0$$

$$\frac{\delta}{\delta a_1} D = -2E\left[\left(x[n] - \sum_{i=1}^{N} a_i \hat{x}[n-i]\right) x[n-1]\right] = 0$$

$$\frac{\delta}{\delta a_2} D = -2E\left[\left(x[n] - \sum_{i=1}^{N} a_i \hat{x}[n-i]\right) x[n-2]\right] = 0$$

$$\vdots$$

$$\frac{\delta}{\delta a_N} D = -2E\left[\left(x[n] - \sum_{i=1}^{N} a_i \hat{x}[n-i]\right) x[n-N]\right] = 0$$

# Optimal Linear Prediction

$$E\left[x[n]x[n-1]\right]=\sum_{i=1}^{N}a_iE\left[x[n-i]x[n-1]\right]\;\Rightarrow R_{xx}(1)=\sum_{i=1}^{N}a_iR_{xx}(i-1)$$

$$E\left[x[n]x[n-2]\right]=\sum_{i=1}^{N}a_iE\left[x[n-i]x[n-2]\right]\Rightarrow R_{xx}(2)=\sum_{i=1}^{N}a_iR_{xx}(i-2)$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$E\left[x[n]x[n-N]\right]=\sum_{i=1}^{N}a_iE\left[x[n-i]x[n-N]\right]$$

$$\Rightarrow R_{xx}(N)=\sum_{i=1}^{N}a_iR_{xx}(i-N)$$

# Optimal Linear Prediction

$$\mathbf{R}_{\mathbf{xx}}\mathbf{a} = \mathbf{p} \quad \Rightarrow \quad \boxed{\mathbf{a} = \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{p}}$$

approximated from time averaging

# Linear Signal Representation

input signal

transform coefficient

basis function

Synthesis
$$\mathbf{x} = \sum_{i=0}^{N-1} y_i \hat{\mathbf{\Phi}}_i$$

Analysis
$$y_i = \langle \mathbf{x}, \mathbf{\Phi}_i \rangle = \mathbf{\Phi}_i^T \mathbf{x}$$

Approximation
$$\hat{\mathbf{x}} = \sum_{i=0}^{N-1} \hat{y}_i \hat{\mathbf{\Phi}}_i$$

using as few coefficients as possible

# Transcform Fundamentals

Analysis Synthesis

$$\mathbf{x} \longrightarrow \boxed{\mathbf{T_A}} \xrightarrow{\mathbf{y}} \boxed{\mathrm{Q}} \xrightarrow{\hat{\mathbf{y}}} \boxed{\mathbf{T_S}} \xrightarrow{\hat{\mathbf{x}}}$$

$$y_i = \langle \mathbf{\Phi_i}, \mathbf{x} \rangle = \mathbf{\Phi}_i^T \mathbf{x}$$

- 1D Analysis Transform

$$\mathbf{y} = \mathbf{T_A x}$$

$\{\mathbf{\Phi_i}\}$ Analysis basis functions

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} = \underbrace{\begin{bmatrix} \text{---} & \mathbf{\Phi_0^T} & \text{---} \\ \text{---} & \mathbf{\Phi_1^T} & \text{---} \\ & \vdots & \\ \text{---} & \mathbf{\Phi_{N-1}^T} & \text{---} \end{bmatrix}}_{\mathbf{T_A}} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

- 1D Synthesis Transform

$$\hat{\mathbf{x}} = \mathbf{T_S}\hat{\mathbf{y}}$$

$\{\hat{\mathbf{\Phi}}_\mathbf{i}\}$ Synthesis basis functions

$$\begin{bmatrix} \hat{x}_0 \\ \hat{x}_1 \\ \vdots \\ \hat{x}_{N-1} \end{bmatrix} = \underbrace{\begin{bmatrix} | & | & & | \\ \hat{\mathbf{\Phi}}_\mathbf{0} & \hat{\mathbf{\Phi}}_\mathbf{1} & \cdots & \hat{\mathbf{\Phi}}_\mathbf{N-1} \\ | & | & & | \end{bmatrix}}_{\mathbf{T_S}} \begin{bmatrix} \hat{y}_0 \\ \hat{y}_1 \\ \vdots \\ \hat{y}_{N-1} \end{bmatrix}$$

# Invertibility & Unitary

- Invertibility
  - perfect reconstruction, bi-orthogonal, reversible

$$\mathbf{T_S} = \mathbf{T_A^{-1}} \Rightarrow \mathbf{T_S T_A} = \mathbf{T_A T_S} = \mathbf{I}$$

$$\left\langle \mathbf{\Phi_i}, \hat{\mathbf{\Phi}}_\mathbf{j} \right\rangle = \delta[i-j]$$

- Unitary
  - orthogonal, orthonormal

$$\mathbf{T_S} = \mathbf{T_A^{-1}} = \mathbf{T_A^T} \Rightarrow \mathbf{T_A^T T_A} = \mathbf{T_A T_A^T} = \mathbf{I}$$

$$\left\langle \mathbf{\Phi_i}, \mathbf{\Phi_j} \right\rangle = \delta[i-j]$$

same analysis & synthesis basis functions

# Norm Preservation

- Norm preservation property of orthonormal transform

$$\left\|\mathbf{y}-\hat{\mathbf{y}}\right\|_2 = \left\|\mathbf{x}-\hat{\mathbf{x}}\right\|_2 \ ?$$

- From a coding perspective
  - Q error in the transform domain equals Q error in the spatial domain!
  - Concentrate on the quantization of the transform coefficients

# 2D Separable Transformation

- 2D Analysis

transforming rows

$$\underset{N \times N}{\mathbf{y}} = \mathbf{T_A}\,\mathbf{x}\,\mathbf{T_A^T}$$

transforming columns

- 2D Synthesis

$$\underset{N \times N}{\mathbf{x}} = \mathbf{T_S}\,\mathbf{y}\,\mathbf{T_S^T} = \mathbf{T_S}\,\mathbf{T_A}\,\mathbf{x}\,\mathbf{T_A^T}\mathbf{T_S^T}$$

- 2D Orthogonal Synthesis

$$\mathbf{x} = \mathbf{T_A^T}\,\mathbf{y}\,\mathbf{T_A} = \mathbf{T_A^T}\,\mathbf{T_A}\,\mathbf{x}\,\mathbf{T_A^T}\mathbf{T_A}$$

# Example

- Discrete Fourier Transform (DFT)

$$F[k] = \sum_{n=0}^{N-1} f[n] W_N^{nk}$$

$$f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] W_N^{-nk}$$

$$n, k \in \{0, 1, \ldots, N-1\} \qquad W_N \equiv e^{-j\frac{2\pi}{N}}$$

$$\mathbf{T_A} = \left\{ W_N^{nk} \right\} \qquad N = 4 \qquad \mathbf{T_A} = \begin{array}{cc} & \xrightarrow{\;\;n\;\;} \\ \Big\downarrow k & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \end{array}$$

$$\mathbf{T_S} = \frac{1}{N} \mathbf{T_A^H}$$

# KLT: Optimal Linear Transform

- Karhunen-Loeve Transform (KLT)
  - Hotelling transform, principle component analysis (PCA)
- Question:
  - Amongst the linear transforms, which one is the best decorrelator, offering the best "compression"?
- Problem:

  Given an $N$-point signal $\mathbf{x}$. Find the set of orthonormal basis functions $\{\mathbf{\Phi_i}\}, i \in \{0,1,\ldots,N-1\}$, such that the MSE between the $L$-point truncated representation $\hat{\mathbf{x}} = \sum_{i=0}^{L-1} y_i \mathbf{\Phi_i}$ $(L < N)$ and the original signal $\mathbf{x}$ is minimized.

- Assumptions: $\mathbf{x}$ is zero-mean WSS RP.

# KLT

- Reminder: $y_i = \langle \mathbf{\Phi_i}, \mathbf{x} \rangle = \mathbf{\Phi_i^T x} = \mathbf{x^T \Phi_i}$
  - Orthonormal constraint: $\langle \mathbf{\Phi_i}, \mathbf{\Phi_j} \rangle = \mathbf{\Phi_i^T \Phi_j} = \delta[i-j]$
  - Autocorrelation matrix

$$\mathbf{R}_{xx} = E\left[\mathbf{x}\mathbf{x}^T\right] = \begin{bmatrix} R_{xx}(0) & R_{xx}(1) & \cdots & R_{xx}(N-1) \\ R_{xx}(1) & R_{xx}(0) & & R_{xx}(N-2) \\ \vdots & \vdots & \ddots & \vdots \\ R_{xx}(N-1) & R_{xx}(N-2) & \cdots & R_{xx}(0) \end{bmatrix}$$

$$MSE \equiv E\left[\left\|\mathbf{x} - \hat{\mathbf{x}}\right\|^2\right] = E\left[\left\|\sum_{i=0}^{N-1} y_i \mathbf{\Phi}_i - \sum_{i=0}^{L-1} y_i \mathbf{\Phi}_i\right\|^2\right] = E\left[\left\|\sum_{i=L}^{N-1} y_i \mathbf{\Phi}_i\right\|^2\right]$$

$$= E\left[\left\langle \sum_{i=L}^{N-1} y_i \mathbf{\Phi}_i, \sum_{j=L}^{N-1} y_j \mathbf{\Phi}_j \right\rangle\right] = E\left[\left(\sum_{i=L}^{N-1} y_i \mathbf{\Phi}_i^T\right)\left(\sum_{j=L}^{N-1} y_j \mathbf{\Phi}_j\right)\right]$$

$$= E\left[\sum_{i=L}^{N-1} \left|y_i\right|^2\right] = E\left[\sum_{i=L}^{N-1} \mathbf{\Phi_i^T x x^T \Phi_i}\right] = \sum_{i=L}^{N-1} \mathbf{\Phi_i^T} E\left[\mathbf{x x^T}\right] \mathbf{\Phi_i}$$

# KLT

$$\frac{\delta}{\delta \mathbf{v}}\left[\mathbf{u}^{\mathrm{T}}\mathbf{v}\right] = \mathbf{u}$$

$$\frac{\delta}{\delta \mathbf{v}}\left[\mathbf{v}^{\mathrm{T}}\mathbf{A}\mathbf{v}\right] = 2\mathbf{A}\mathbf{v}$$

- Reminder: $\underbrace{\mathbf{R}_{\mathbf{xx}}\mathbf{e}}_{\text{eigenvector}} = \overbrace{\lambda}^{\text{eigenvalue}}\mathbf{e}$

$$\text{Minimize } MSE = \sum_{i=L}^{N-1} \mathbf{\Phi}_{\mathbf{i}}^{\mathbf{T}} E\left[\mathbf{x}\mathbf{x}^{\mathbf{T}}\right]\mathbf{\Phi}_{\mathbf{i}} = \sum_{i=L}^{N-1} \mathbf{\Phi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{R}_{\mathbf{xx}} \mathbf{\Phi}_{\mathbf{i}}$$

$$\text{wrt } \left\langle \mathbf{\Phi}_{\mathbf{i}}, \mathbf{\Phi}_{\mathbf{j}} \right\rangle = \delta[i-j]$$

- Lagrange Multiplier

$$\frac{\delta}{\delta \mathbf{\Phi}_{\mathbf{i}}}\left[\sum_{i=L}^{N-1} \mathbf{\Phi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{R}_{\mathbf{xx}} \mathbf{\Phi}_{\mathbf{i}} - \lambda_i \left(\left\langle \mathbf{\Phi}_{\mathbf{i}}, \mathbf{\Phi}_{\mathbf{j}} \right\rangle - 1\right)\right] = 0$$

$$\frac{\delta}{\delta \mathbf{\Phi}_{\mathbf{i}}}\left[\mathbf{\Phi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{R}_{\mathbf{xx}} \mathbf{\Phi}_{\mathbf{i}} - \lambda_i \left(\left\langle \mathbf{\Phi}_{\mathbf{i}}, \mathbf{\Phi}_{\mathbf{i}} \right\rangle - 1\right)\right] = 0$$

$$2\mathbf{R}_{\mathbf{xx}}\mathbf{\Phi}_{\mathbf{i}} - 2\lambda_i \mathbf{\Phi}_{\mathbf{i}} = 0 \Rightarrow \boxed{\mathbf{R}_{\mathbf{xx}}\mathbf{\Phi}_{\mathbf{i}} = \lambda_i \mathbf{\Phi}_{\mathbf{i}}} \qquad \Rightarrow MSE = \sum_{i=L}^{N-1} \lambda_i$$

- Optimal coding scheme: send the larger eigenvalues first!

# KLT Problems

- KLT problems
  - Signal dependent
  - Computationally expensive
    - statistics need to be computed
    - no structure, no symmetry, no guarantee of stability
  - Real signals are really stationary
  - Encoder/Decoder communication
- Practical solutions
  - Assume a reasonable signal model
  - Blocking the signals to ensure stationary assumption holds
  - Making the transform matrix sparse & symmetric
  - Good KLT approximation for smooth signals: DCT!

# KLT: Summary

$$\mathbf{R_{xx}\Phi_i} = \lambda_i \mathbf{\Phi_i}$$

eigenvectors

$$E\left[\mathbf{xx}^T\right] \qquad KLT = \begin{bmatrix} | & | & & | \\ \mathbf{\Phi_0} & \mathbf{\Phi_1} & \cdots & \mathbf{\Phi_{N-1}} \\ | & | & & | \end{bmatrix}$$

- Signal dependent
- Require stationary signals
- How do we communicate bases to the decoder?
- How do we design “good” signal-independent transform?

# Best Basis Revisited

- Fundamental question: what is the best basis?
  - energy compaction: minimize a pre-defined error measure, say MSE, given $L$ coefficients
  - maximize perceptual reconstruction quality
  - low complexity: fast-computable decomposition and reconstruction
  - intuitive interpretation
- How to construct such a basis? Different viewpoints!
- Applications
  - compression, coding
  - signal analysis
  - de-noising, enhancement
  - communications

# Discrete Cosine Transforms

- Type I

$$K_i = \begin{cases} 1/\sqrt{2}, & i = 0, M \\ 1, & \text{otherwise} \end{cases}$$

$$\left[C^I\right] = \sqrt{\frac{2}{M}}\left[K_m K_n \cos\left(\frac{mn\pi}{M}\right)\right], \quad m,n \in \{0,1,\ldots,M\}$$

- Type II

$$\boxed{\left[C^{II}\right] = \sqrt{\frac{2}{M}}\left[K_m \cos\left(\frac{m(n+1/2)\pi}{M}\right)\right], \quad m,n \in \{0,1,\ldots,M-1\}}$$

- Type III

$$\left[C^{III}\right] = \sqrt{\frac{2}{M}}\left[K_n \cos\left(\frac{(m+1/2)n\pi}{M}\right)\right], \quad m,n \in \{0,1,\ldots,M-1\}$$

- Type IV

$$\left[C^{IV}\right] = \sqrt{\frac{2}{M}}\left[\cos\left(\frac{(m+1/2)(n+1/2)\pi}{M}\right)\right], \quad m,n \in \{0,1,\ldots,M-1\}$$

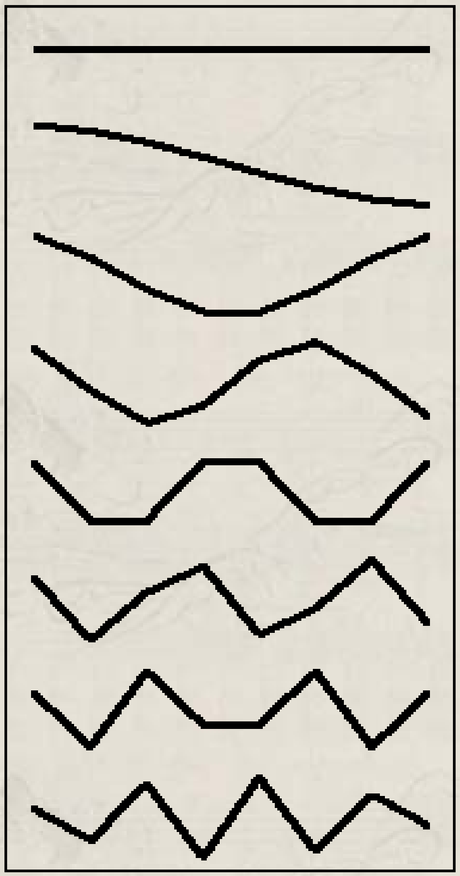
# DCT Type-II

$$
\begin{cases}
X[m] = \sqrt{\dfrac{2}{M}} K_m \displaystyle\sum_{n=0}^{M-1} x[n] \cos\left[\dfrac{(2n+1)m\pi}{2M}\right] \\
x[n] = \sqrt{\dfrac{2}{M}} K_n \displaystyle\sum_{m=0}^{M-1} X[m] \cos\left[\dfrac{(2m+1)n\pi}{2M}\right]
\end{cases}
$$

$$m, n = 0,1,\ldots, M-1$$

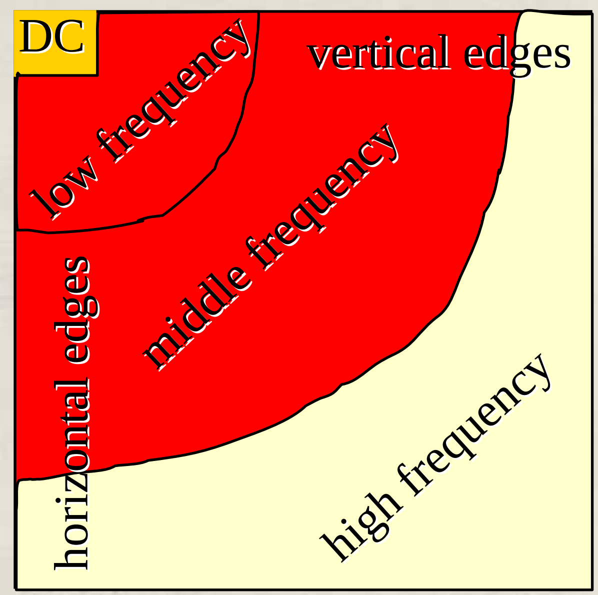
$$
K_i = \begin{cases} \dfrac{1}{\sqrt{2}}, & i = 0 \\ 1, & i \neq 0 \end{cases}
$$

DCT basis



- orthogonal
- real coefficients
- symmetry
- near-optimal
- fast algorithms

8 x 8 block



# DCT Symmetry

$$\cos\left(\frac{m(2(M-1-n)+1)\pi}{2M}\right)$$

$$=\cos\left(\frac{(2M-2-2n+1)m\pi}{2M}\right)$$

$$=\cos\left[\frac{2Mm\pi}{2M}-\frac{(2n+1)m\pi}{2M}\right]$$

$$=\pm\cos\left[\frac{(2n+1)m\pi}{2M}\right]$$

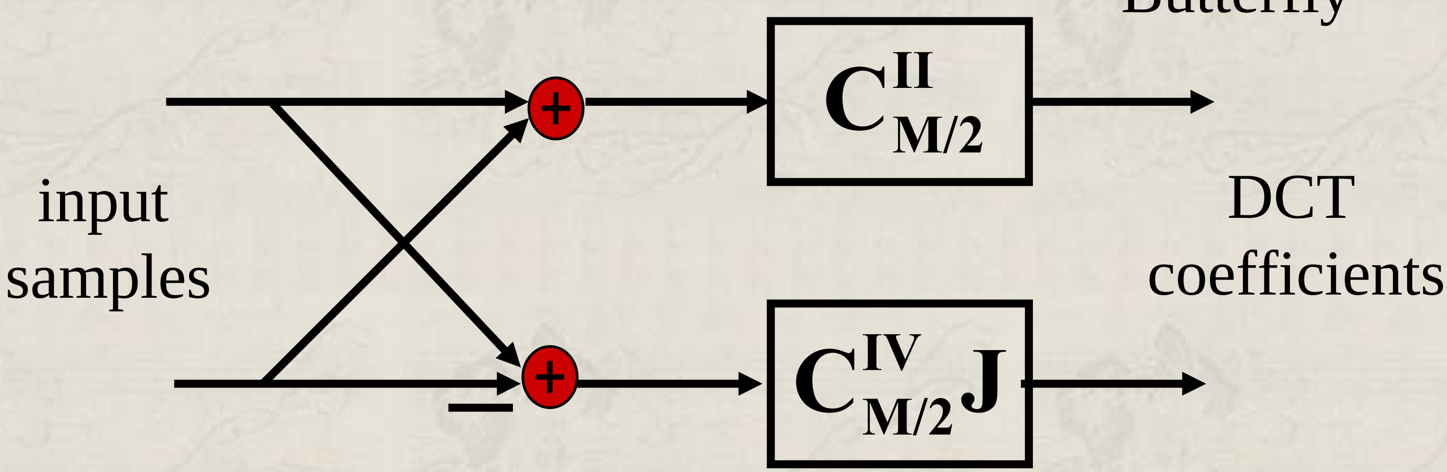
DCT basis functions are either symmetric or anti-symmetric

# DCT: Recursive Property

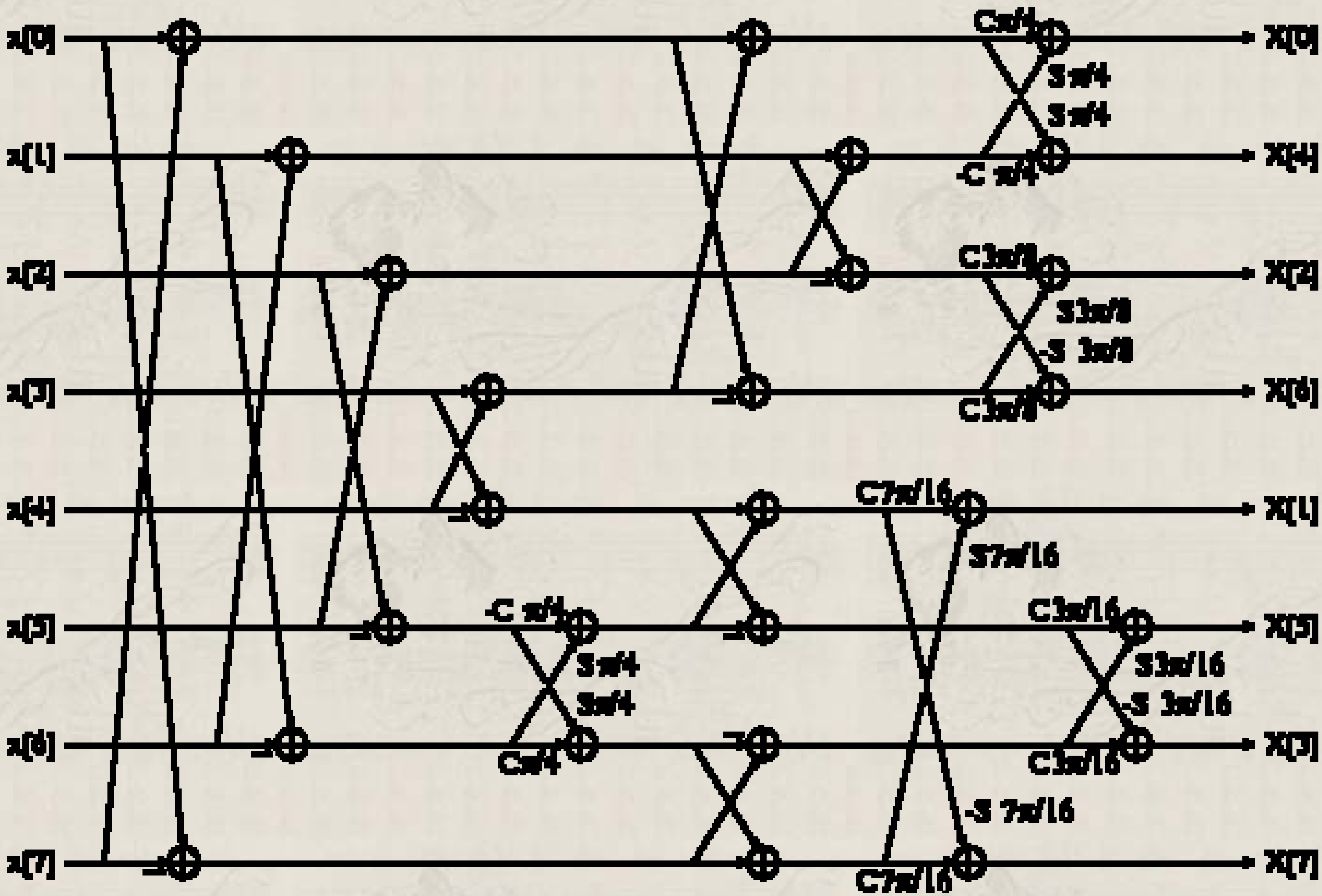
- An M-point DCT–II can be implemented via an M/2-point DCT–II and an M/2-point DCT–IV

$$\left[\mathbf{C}_{\mathbf{M}}^{\mathbf{II}}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} \boxed{\mathbf{C}_{\mathbf{M/2}}^{\mathbf{II}}} & \mathbf{0} \\ \mathbf{0} & \boxed{\mathbf{C}_{\mathbf{M/2}}^{\mathbf{IV}}}\mathbf{J} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{J} \\ \mathbf{J} & -\mathbf{I} \end{bmatrix}$$

Butterfly



# Fast DCT Implementation




13 multiplications and 29 additions per 8 input samples

# Block DCT

$$\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_N \end{bmatrix} = \begin{bmatrix} \mathbf{C}_M^{II} & \mathbf{0} & & \\ \mathbf{0} & \mathbf{C}_M^{II} & \mathbf{0} & \\ & \mathbf{0} & \mathbf{C}_M^{II} & \mathbf{0} \\ & & \mathbf{0} & \mathbf{C}_M^{II} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix}$$

output blocks
of DCT coefficients,
each of size M

input blocks,
each of size M

# JPEG Still Image Coding Standard

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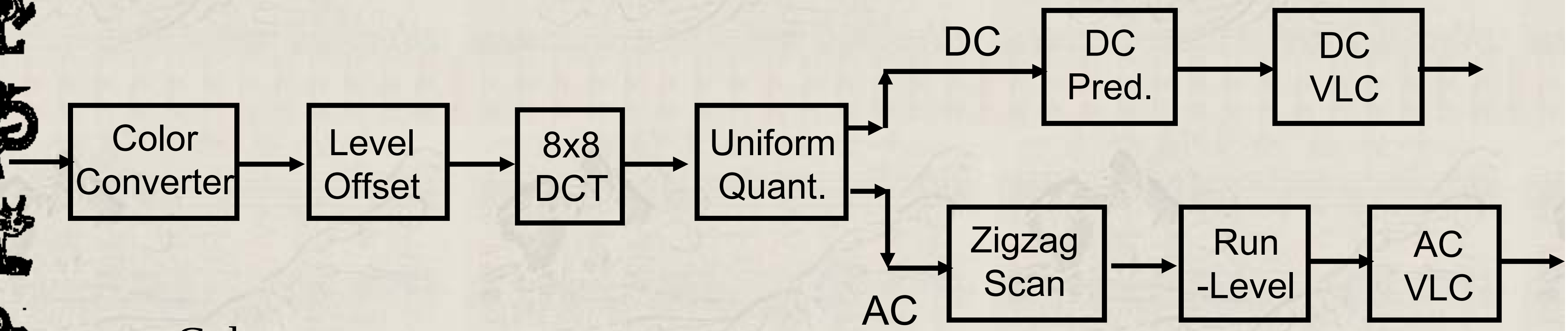
# Overall Structure of JPEG




- Color converter
  - RGB to YUV
- Level offset
  - subtract 2^(N-1). N: bits / pixel.
- Quantization
  - Different step size for different coefficients
- DC
  - Predict from DC of previous block
- AC:
  - Zigzag scan to get 1-D data
  - Run-level: joint coding of non-zero coeffs and number of zeros before it.

# JPEG Quantization

- Uniform mid-tread quantizer
- Larger step sizes for chroma components
- Different coefficients have different step sizes
  - Smaller steps for low frequency coefficients (more bits)
  - Larger steps for high frequency coefficients (less bits)
  - Human visual system is not sensitive to error in high frequency

- Luma Quantization Table

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 11 | 10 | 16 | 24 | 40 | 51 | 51 |
| 12 | 12 | 14 | 19 | 26 | 58 | 60 | 55 |
| 14 | 13 | 16 | 24 | 40 | 57 | 69 | 56 |
| 14 | 17 | 22 | 29 | 51 | 87 | 80 | 62 |
| 18 | 22 | 37 | 56 | 68 | 109 | 103 | 77 |
| 24 | 35 | 55 | 64 | 81 | 104 | 113 | 92 |
| 49 | 64 | 78 | 87 | 103 | 121 | 120 | 101 |
| 72 | 92 | 95 | 98 | 112 | 100 | 103 | 99 |

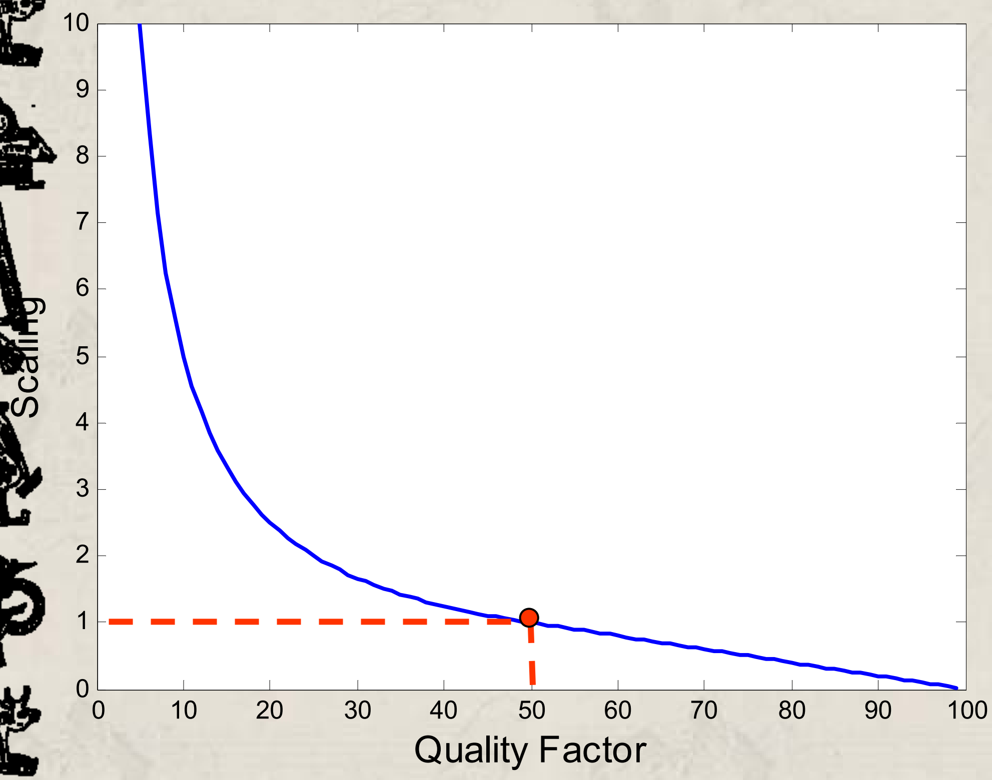
- Chroma Quantization Table

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 17 | 18 | 24 | 47 | 99 | 99 | 99 | 99 |
| 18 | 21 | 26 | 66 | 99 | 99 | 99 | 99 |
| 24 | 26 | 56 | 99 | 99 | 99 | 99 | 99 |
| 47 | 66 | 99 | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |

- Actual step size: Scale the basic table by a quality factor

# Scaling of Quantization Table

- Actual Q table = scaling x Basic Q table:
  - quality factor ≤ 50: scaling = 50/quality
  - quality factor > 50: scaling = 2 - quality/50

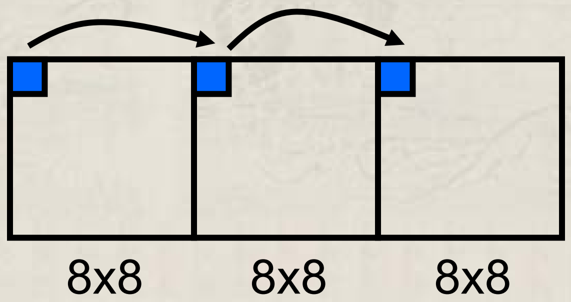



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 11 | 10 | 16 | 24 | 40 | 51 | 51 |
| 12 | 12 | 14 | 19 | 26 | 58 | 60 | 55 |
| 14 | 13 | 16 | 24 | 40 | 57 | 69 | 56 |
| 14 | 17 | 22 | 29 | 51 | 87 | 80 | 62 |
| 18 | 22 | 37 | 56 | 68 | 109 | 103 | 77 |
| 24 | 35 | 55 | 64 | 81 | 104 | 113 | 92 |
| 49 | 64 | 78 | 87 | 103 | 121 | 120 | 101 |
| 72 | 92 | 95 | 98 | 112 | 100 | 103 | 99 |

| Quality Factor | Scaling |
|---|---|
| 10 | 5.0 |
| 20 | 2.5 |
| 50 | 1.0 |
| 75 | 0.5 |

# DC Prediction

- DC Coefficients: average of a block
- DC of neighboring blocks are still similar to each others: redundancy
- The redundancy can be removed by differential coding:
  - $e(n) = DC(n) - DC(n-1)$
- Only encode the prediction error *e(n)*




DC coeffs
of Lena

# Coefficient Category

- Divide coefficients into categories of exponentially increased sizes
- Use Huffman code to encode category ID
- Use fixed length code within each category
- Similar to Exponential Golomb code

| Ranges | Range Size | DC Cat. ID | AC Cat. ID |
|---|---|---|---|
| 0 | 1 | 0 | N/A |
| -1, 1 | 2 | 1 | 1 |
| -3, -2, 2, 3 | 4 | 2 | 2 |
| -7, -6, -5, -4, 4, 5, 6, 7 | 8 | 3 | 3 |
| -15, …, -8, 8, …, 15 | 16 | 4 | 4 |
| -31, …, -16, 16, …, 31 | 32 | 5 | 5 |
| -63, …, -32, 32, …, 63 | 64 | 6 | 6 |
| … | … | … | … |
| **[-32767, -16384], [16384, 32767]** | 32768 | 15 | 15 |

# Coding of DC Coefficients

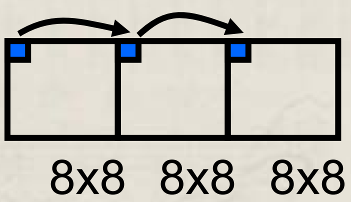
- Encode $e(n) = DC(n) – DC(n-1)$

| DC Cat. | Prediction Errors | Base Codeword |
|---|---|---|
| 0 | 0 | 010 |
| 1 | -1, 1 | 011 |
| 2 | -3, -2, 2, 3 | 100 |
| 3 | -7, -6, -5, -4, 4, 5, 6, 7 | 00 |
| 4 | -15, …, -8, 8, …, 15 | 101 |
| 5 | -31, …, -16, 16, …, 31 | 110 |
| 6 | -63, …, -32, 32, …, 63 | 1110 |
| … | … | … |

Our example:

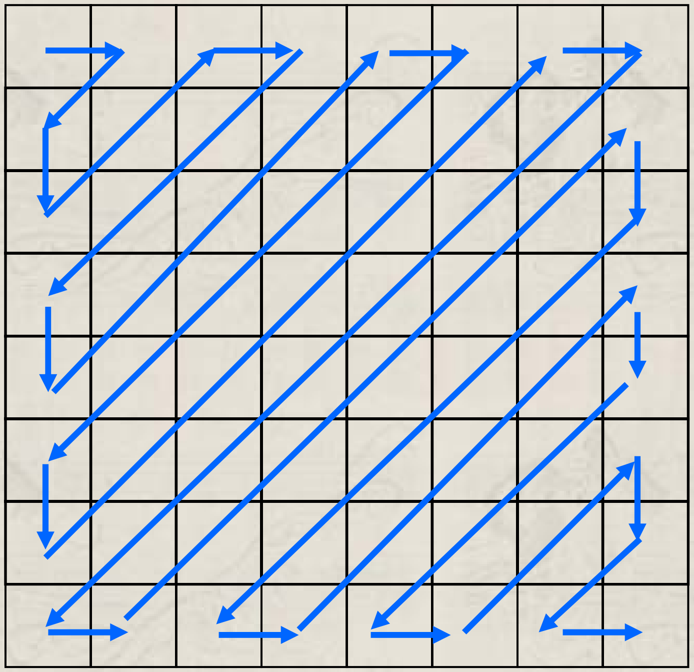
DC: 8. Assume last DC: 5 ➔ e = 8 – 5 = 3.

Cat.: 2, index 3 ➔ Bitstream: 10011



# Coding of AC Coefficients

- Most non-zero coefficients are in the upper-left corner
- Zigzag scanning



- Example

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8 | 24 | -2 | 0 | 0 | 0 | 0 | 0 |
| -31 | -4 | 6 | -1 | 0 | 0 | 0 | 0 |
| 0 | -12 | -1 | 2 | 0 | 0 | 0 | 0 |
| 0 | 0 | -2 | -1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

- Zigzag scanning result (DC is coded separately):

  24 -31 0 -4 -2 0 6 -12 0 0 0 -1 -1 0 0 0 2 -2 0 0 0 0 0 -1 EOB

  EOB: End of block symbol. The remaining coeffs are all 0.

# Coding of AC Coefficients

- Many AC coefficients are zeros:
  - Huffman coding is not efficient for symbol with prob. > 1/2
- Run-level coding: Jointly encode a non-zero coefficient and the number of zeros before it (run of zeros): (run, level) event
- Disadvantage: Symbol set is enlarged: #Run x #Level
- Tradeoff:
  - Run: encode up to 15 zeros. Apply escape coding for greater values.
  - Level: Divide level into 16 categories, as in DC.
  - Apply Huffman coding to the joint Run / Category event:
    - Max symbol set size: 16 x 16 = 256.
  - Followed by fixed length code to signal the level index within each category
- Example: zigzag scanning result

  24 -31 0 -4 -2 0 6 -12 0 0 0 -1 -1 0 0 0 2 -2 0 0 0 0 0 -1 EOB
- (Run, level) representation:
- (0, 24), (0, -31), (1, -4), (0, -2), (1, 6), (0, -12), (3, -1), (0, -1), (3, 2), (0, -2), (5, -1), EOB

# Coding of AC Coefficients

| Run / Cat. | Base codeword | Run / Cat. | Base Codeword | … | Run / Cat. | Base codeword |
|---|---|---|---|---|---|---|
| EOB | 1010 | - | - | … | ZRL | 1111 1111 001 |
| 0/1 | 00 | 1/1 | 1100 | … | 15/1 | 1111 1111 1111 0101 |
| 0/2 | 01 | 1/2 | 11011 | … | 15/2 | 1111 1111 1111 0110 |
| 0/3 | 100 | 1/3 | 1111001 | … | 15/3 | 1111 1111 1111 0111 |
| 0/4 | 1011 | 1/4 | 111110110 | … | 15/4 | 1111 1111 1111 1000 |
| 0/5 | 11010 | 1/5 | 11111110110 | … | 15/5 | 1111 1111 1111 1001 |
| … | … | … | … | … | … | … |

- ZRL: represent 16 zeros when number of zeros exceeds 15.
  - Example: 20 zeros followed by -1: (ZRL), (4, -1).

- (Run, Level) sequence: (0, 24), (0, -31), (1, -4), ……
- Run/Cat. Sequence: 0/5, 0/5, 1/3, …

24 is the 24-th entry in Category 5 ➔ (0, 24): 11010 <u>11000</u>

-4 is the 3-th entry in Category 3 ➔ (1, -4): 1111001 <u>011</u>

# A Complete Example

- Original data: 2-D DCT

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 124 | 125 | 122 | 120 | 122 | 119 | 117 | 118 |
| 121 | 121 | 120 | 119 | 119 | 120 | 120 | 118 |
| 126 | 124 | 123 | 122 | 121 | 121 | 120 | 120 |
| 124 | 124 | 125 | 125 | 126 | 125 | 124 | 124 |
| 127 | 127 | 128 | 129 | 130 | 128 | 127 | 125 |
| 143 | 142 | 143 | 142 | 140 | 139 | 139 | 139 |
| 150 | 148 | 152 | 152 | 152 | 152 | 150 | 151 |
| 156 | 159 | 158 | 155 | 158 | 158 | 157 | 156 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 39.8 | 6.5 | -2.2 | 1.2 | -0.3 | -1.0 | 0.7 | 1.1 |
| -102.4 | 4.5 | 2.2 | 1.1 | 0.3 | -0.6 | -1.0 | -0.4 |
| 37.7 | 1.3 | 1.7 | 0.2 | -1.5 | -2.2 | -0.1 | 0.2 |
| -5.6 | 2.2 | -1.3 | -0.8 | 1.4 | 0.2 | -0.1 | 0.1 |
| -3.3 | -0.7 | -1.7 | 0.7 | -0.6 | -2.6 | -1.3 | 0.7 |
| 5.9 | -0.1 | -0.4 | -0.7 | 1.9 | -0.2 | 1.4 | 0.0 |
| 3.9 | 5.5 | 2.3 | -0.5 | -0.1 | -0.8 | -0.5 | -0.1 |
| -3.4 | 0.5 | -1.0 | 0.8 | 0.9 | 0.0 | 0.3 | 0.0 |

- Quantized by basic table

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| -9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Q table:

| | | |
|---|---|---|
| 16 | 11 | … |
| 12 | … | |
| 14 | … | |

floor(39.8/16 + 0.5) = 2
floor(6.5/11 + 0.5) = 1
-floor(102.4/12 + 0.5) = -9
floor(37.7/14 + 0.5) = 3

- Zigzag scanning

2 1 -9 3 EOB

# A Complete Example

- Zigzag scanning

  2 1 -9 3 EOB

- Inverse Quantization

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 32 | 11 | 0 | 0 | 0 | 0 | 0 | 0 |
| -108 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 42 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

- Reconstructed block

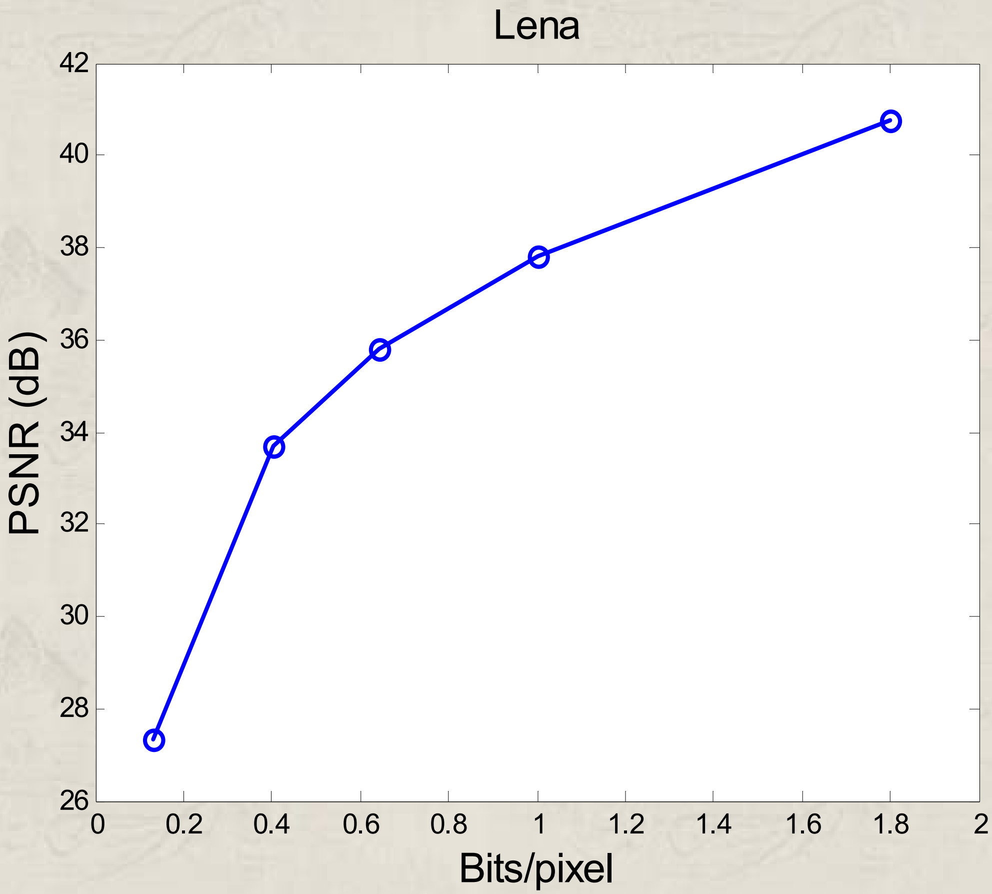
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 122 | 122 | 121 | 121 | 120 | 119 | 119 | 118 |
| 121 | 121 | 120 | 119 | 119 | 118 | 117 | 117 |
| 120 | 120 | 120 | 119 | 118 | 117 | 117 | 117 |
| 123 | 123 | 122 | 122 | 121 | 120 | 120 | 120 |
| 131 | 130 | 130 | 129 | 128 | 128 | 127 | 127 |
| 142 | 141 | 141 | 140 | 139 | 139 | 138 | 138 |
| 153 | 152 | 152 | 151 | 150 | 150 | 149 | 149 |
| 159 | 159 | 159 | 158 | 157 | 157 | 156 | 156 |

- MSE: 5.67

# Progressive JPEG

- Baseline JPEG encodes the image block by block:
  - Decoder has to wait till the end to decode and display the entire image
- Progressive: Coding DCT coefficients in multiple scans
  - The first scan generates a low-quality version of the entire image
  - Subsequent scans refine the entire image gradually.
- Two procedures defined in JPEG:
  - Spectral selection:
    - Divide all DCT coefficients into several bands (low, middle, high frequency subbands…)
    - Bands are coded into separate scans
  - Successive approximation:
    - Send MSB of all coefficients first
    - Send lower significant bits in subsequent scans

# JPEG Coding Result for Lena




Quality factor:

5 25 50 75 90

QF 25

QF 5

Blocking artifact →

# Scalable Coding & JPEG2000

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# Fundamentals

- Scalability coding: capability of recovering physically meaningful signal information by decoding only partial compressed bit-stream
- Scalable coding generates a single coded representation (bit-stream) in a manner that facilitates the derivation of signal of many different resolutions and qualities at the decoder
- Embedded or progressive bit-stream: a bit stream that can be truncated at any point and the decoded signal is the same as if the signal has been originally encoded at that rate
- Embeddedness is the extreme of scalability, sometimes labeled fine-granularity scalability

# Goals and Approaches

- Simulcast coding
  - Encode the same signal several times, each with a different quality setting
  - Each of the generated bit-stream is non-scalable
  - Advantage: simple, efficient for each particular setting
  - Disadvantage: inefficient overall
- Design goal in scalable coding
  - Realizing requirement for scalability
  - Minimizing the reduction in coding efficiency
- Approach
  - Coarse-granularity scalability: only have a few layers, usually two to three only
  - Fine-granularity scalability: many layers, offer more decoding options and precise bit-rate control

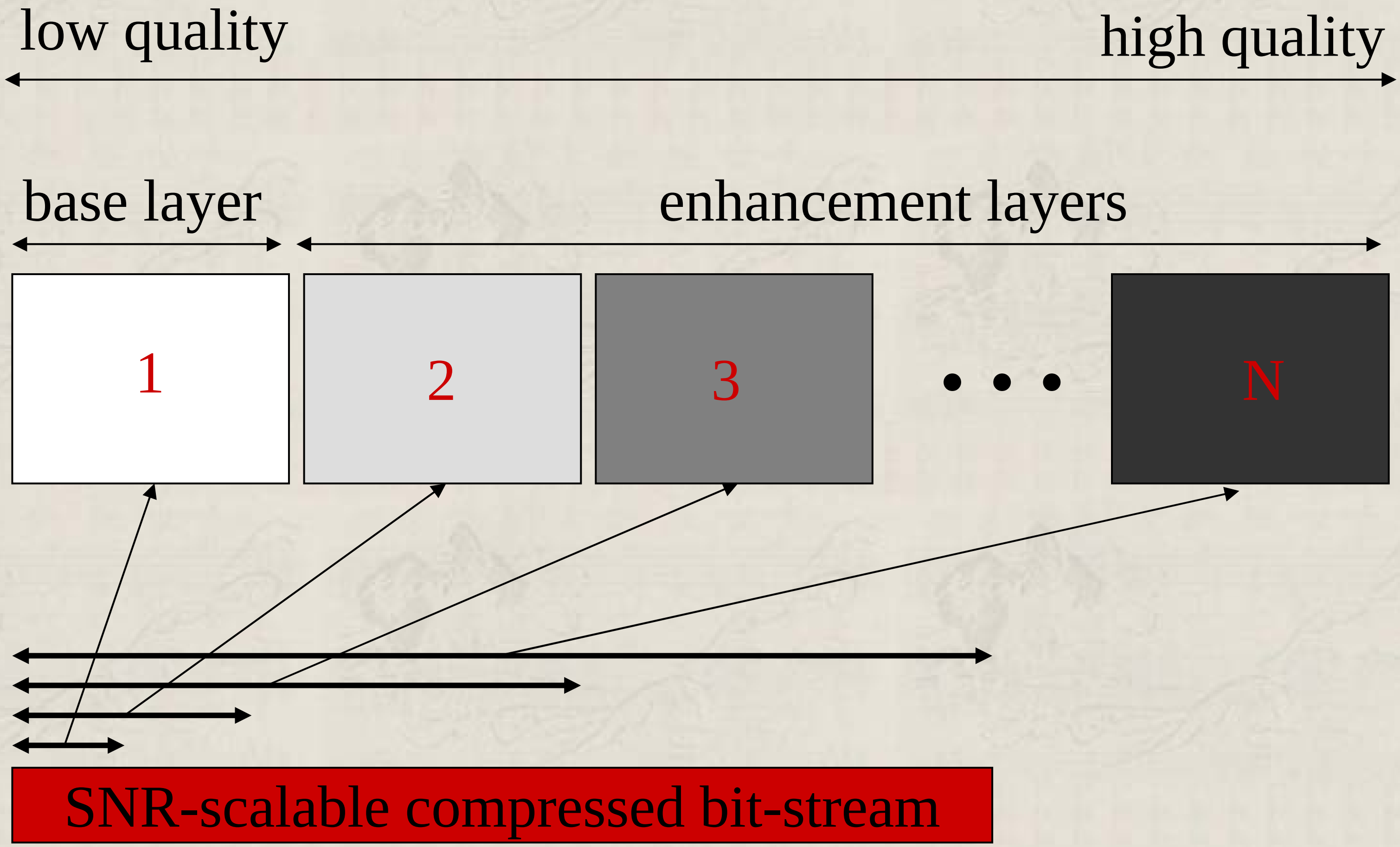
# Scalability Classification

- Quality or SNR scalability
  - Represent signal with many layers, each at a different quality level or at different accuracy
- Spatial scalability
  - More than one layer and they can usually have different spatial resolution
- Temporal scalability
  - More than one layer & each can have different temporal resolution (frame rate)
- Frequency scalability or data partitioning
  - Single-coded bit-stream is artificially partitioned into layers, each contains different frequency content
- Hybrid scalability
  - Combination of two or more types of scalability above

# Scalable Applications

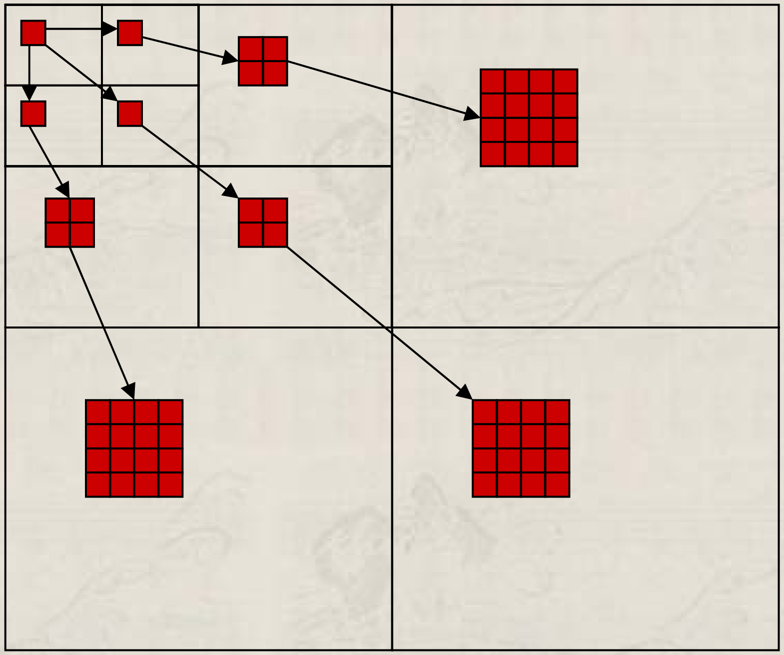
- Quality/SNR scalability
  - Digital broadcast TV or HDTV with different quality layers
  - Multi-quality video-on-demand services
  - Error-resilient video over ATM and other networks
- Spatial scalability
  - Inter-working between two different video standards
  - Layered digital TV broadcast
  - Video on LAN and computer networks
  - Error-resilient video over lossy channels
- Temporal scalability
  - Migration from low to high temporal resolution
  - Networked video. Error resilience
  - Multi-quality video-on-demand services based on decoder capability as well as communication bit-rate
- Frequency scalability
  - Error resilience

# Quality/SNR Scalability




- N layers of quality/SNR scalability

# Wavelet Zero-Tree

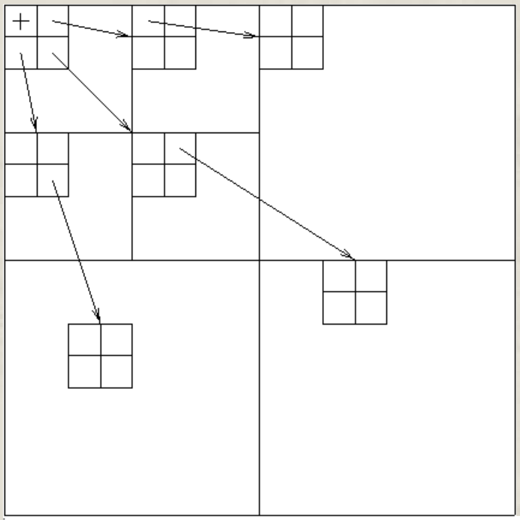


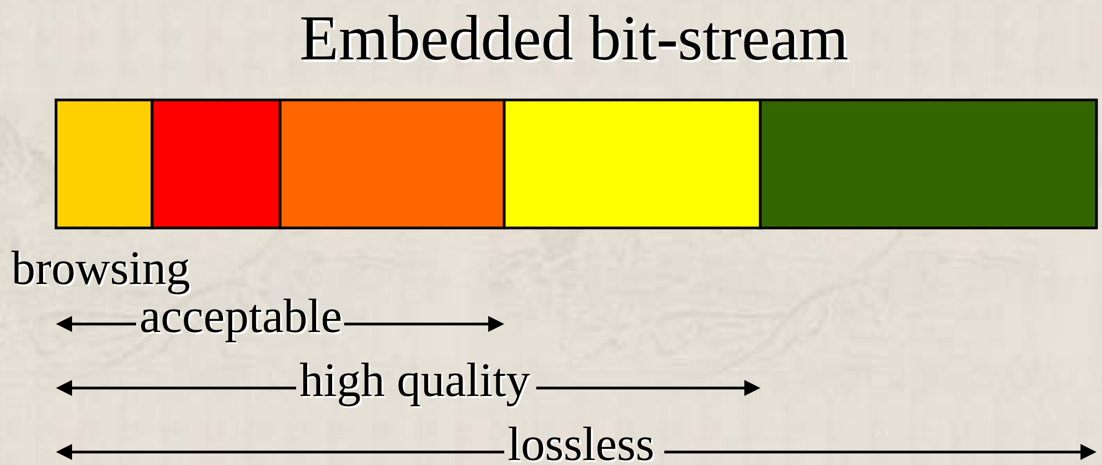
- Main observation: there is self-similarity between wavelet coefficients across different scales
- If a parent is insignificant with respect to a threshold T, i.e. |C| < T, then so are its children

$$\text{parent}: \; c_{i,j}$$

$$\text{children}: \; \{c_{2i,2j}, c_{2i+1,2j}, c_{2i,2j+1}, c_{2i+1,2j+1}\}$$

# Wavelet Bit Plane Coding






| | sign | s | s | s | s | s | s | s | s | s | s | s | s | s |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| msb | 5 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 4 | | | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | | | | | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | 2 | | | | | | | | | 1 | 1 | 1 | 1 | 1 |
| | 1 | | | | | | | | | | | | | |
| lsb | 0 | | | | | | | | | | | | | |

# EZW Coding

- Embedded zero-tree wavelet coding [Shapiro 1993]
  - Wavelet transform for image de-correlation
  - Exploitation of self-similarity of wavelet coefficients across different scales to predict the location of significant information
  - Further compression with adaptive arithmetic coding
- Main features
  - Bit-plane coding
  - One sorting pass and one refinement pass per bit plane with a pre-defined scan pattern
  - Use four symbols to classify wavelet coefficients
    - POS: positive significant
    - NEG: negative significant
    - ZTR: zero-tree root; parent and all children are insignificant
    - IZ: isolated insignificant; parent is insignificant but at least one of the children is significant

# Toy Example

wavelet coefficients

| | | | |
|---|---|---|---|
| 18 | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| 8 | 13 | -6 | 4 |
| -7 | 1 | 3 | -2 |

- Rank coefficients by magnitude
- Transmit coefficients bit plane by bit plane: 0 010 10011100
- Problem: how do we transmit the rank order to the decoder?

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sign | + | + | + | + | + | — | — | + | + | + | + | + | — | — | + | + |
| MSB | 1 | | | | | | | | | | | | | | | |
| | 0 | 1 | 1 | | | | | | | | | | | | | |
| | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | | | | | | | | |
| | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | | |
| LSB | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| | 18 | 13 | 8 | 7 | 6 | -6 | -5 | 4 | 3 | 3 | 2 | 2 | -2 | -2 | 1 | 1 |

# Recall: Embedded Quantization

| Original coefficient $C = 22$ | Truncate 4 bit planes | Receive 1 refinement bit | Receive 2 refinement bits |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 0 | X | 0 | 0 |
| 1 | X | X | 1 |
| 1 | X | X | X |
| 0 | X | X | X |
|  | Range=[16, 32) | Range=[16, 24) | Range=[20, 24) |
|  | Cr = 24 | Cr = 20 = 24 – 4 | Cr = 22 = 20 + 2 |

- $N$-bit-plane truncation = scalar quantization with $\Delta = 2^N$

# EZW Basic Algorithm

- Set initial threshold: $T = 2^{\lfloor \log_2 |\max| \rfloor}$
- Sorting Pass – Dominant Pass
  - scan coefficients from top left corner
  - parent nodes are always scanned before children
  - For each coefficient, output a symbol among {POS, NEG, ZTR, IZ} depending on the threshold $T$
- Refinement Pass – Subordinate Pass
  - refine the accuracy of each significant coefficient by sending one additional bit of its binary representation
- Reduce the threshold by a factor of 2: $T = \frac{1}{2}T$ and repeat Step 2

# EZW Example: First Bit Plane

| | | | |
|---|---|---|---|
| 18 | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| 8 | 13 | -6 | 4 |
| -7 | 1 | 3 | -2 |

POS =11
NEG =10
IZ =01
ZTR =00

- T=16
- Dominant Pass 1
  - POS ZTR ZTR ZTR
  - Subordinate list = {18}
- Subordinate Pass 1
  - No symbols because subordinate step ***i*** works on significant coefficients from dominant step ***i-1*** and earlier

Compressed bit-stream

11 00 00 00 – 8 bits

Reconstruction = {24 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0}

# EZW Example: 2nd Bit Plane

| | | | |
|---|---|---|---|
| * | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| 8 | 13 | -6 | 4 |
| -7 | 1 | 3 | -2 |

POS =11
NEG =10
IZ =01
ZTR =00

- T=8
- Dominant Pass 2
  - ZTR IZ ZTR POS POS IZ IZ
  - Subordinate list = {18 8 13}
- Subordinate Pass 2
  - Send the bit plane of coefficients involved in Dominant Pass 1

Compressed bit-stream

00 01 00 11 11 01 01 – 14 bits

0 – 1 bit

Reconstruction = {20 12 12 0 0 0 0 0 0 0 0 0 0 0 0 0}

Bit budget = 23 bits

# EZW Example: 3rd Bit Plane

| | | | |
|---|---|---|---|
| * | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| * | * | -6 | 4 |
| -7 | 1 | 3 | -2 |

POS =11
NEG =10
IZ =01
ZTR =00

- T=4
- Dominant Pass 3
  - ZTR POS NEG NEG IZ NEG POS IZ IZ
  - Subordinate list = {18,8,13,6,-5,-7,-6,4}
- Subordinate Pass 3
  - Send the bit plane of coefficients involved in Dominant Pass 2

Compressed bit-stream

00 11 10 10 01 10 11 01 01 – 18 bits

001 – 3 bits

Reconstruction = {18 10 14 6 -6 -6 -6 6 0 0 0 0 0 0 0 0}

Bit budget = 44 bits

# EZW Decoding

- The decoder needs
  - Initial threshold T (or the max absolute value of all coefficients)
  - Original image size
  - Number of wavelet decomposition levels
  - Encoded bit-stream
- Decoding process
  - Decode the arithmetic-encoded bit-stream into a stream of symbols
  - Based on the side information, create data structures of appropriate sizes
  - Traverse the encoding algorithm

# Other Approaches

- Idea can be generalized to other different data structures
- For example, quad-tree
- Sorting Pass 1
  - 1 0 0 0 1 0 0 0
- Refinement Pass 1: nothing
- Sorting Pass 2
  - 0 0 1 0 1 1 0 0
- Refinement Pass 2
  - Like EZW, 1 bit for 18
- Sorting Pass 3
  - 1 0 1 1 0 1 1 1 0 1 1 0 0
- Refinement Pass 3
  - Like EZW, 3 bits for 18 8 13

| | | | |
|---|---|---|---|
| 18 | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| 8 | 13 | -6 | 4 |
| -7 | 1 | 3 | -2 |

| | | | |
|---|---|---|---|
| 0 | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| 8 | 13 | -6 | 4 |
| -7 | 1 | 3 | -2 |

| | | | |
|---|---|---|---|
| 0 | 3 | 2 | 2 |
| 6 | -5 | 1 | -2 |
| 0 | 0 | -6 | 4 |
| -7 | 1 | 3 | -2 |

# JPEG2000 Image Coding

- About JPEG2000 (ISO/IEC15444)
- Objectives of JPEG2000
  - To provide new functionalities and features that current standards fail to support
  - To support advanced applications in the new millennium
  - To extend the applicability of image coding in more applications
  - To allow imaging applications to be interactive and adaptive

# JPEG2000 vs. JPEG

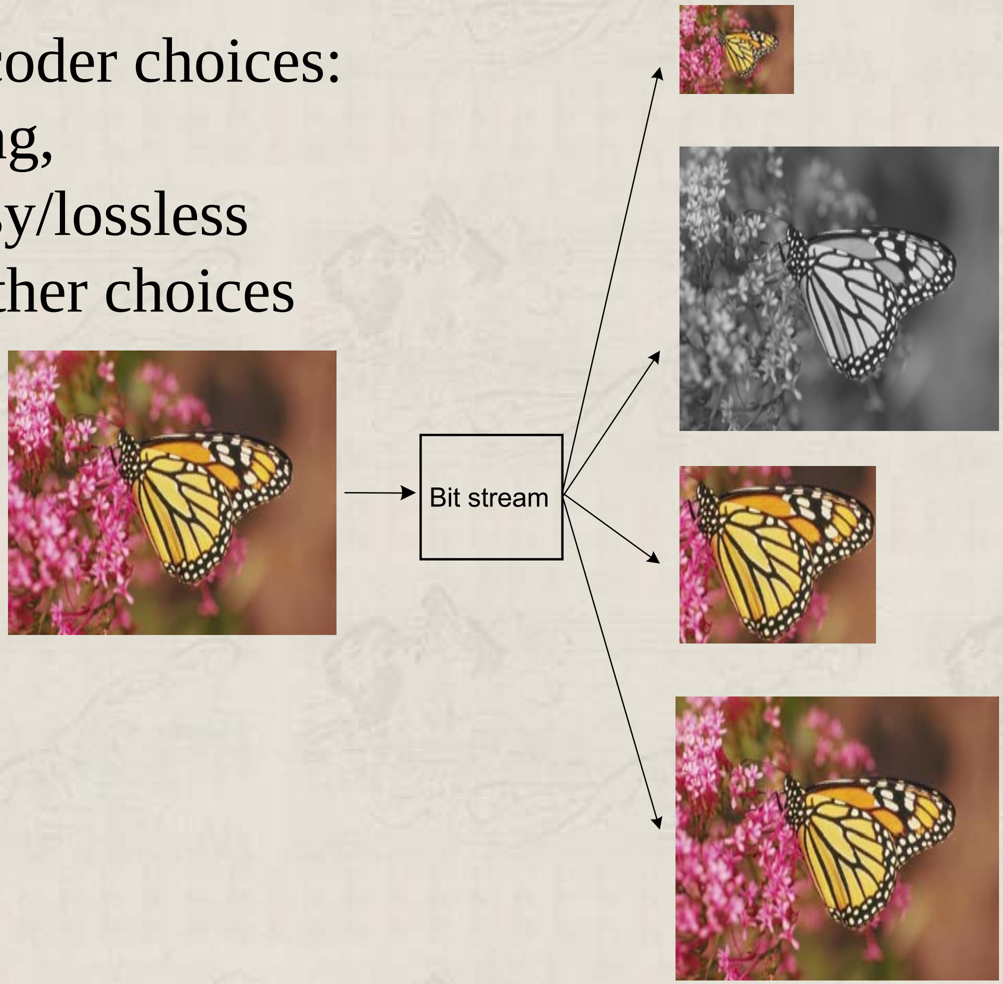
- Key Advantages
  - Wavelet based – better rate-distortion performance
  - Scalable by resolution, quality, color channel, location in image
  - Lossless encoding, including lossy to lossless scalability
  - Error resilience
  - Region-of-Interest coding and progressive decoding

Compression ratio: 100:1


*http://www.aware.com/products/compression/demos/lena_compare.html*

# JPEG2000 Flexible Decoding

Encoder choices:
tiling,
lossy/lossless
+ other choices

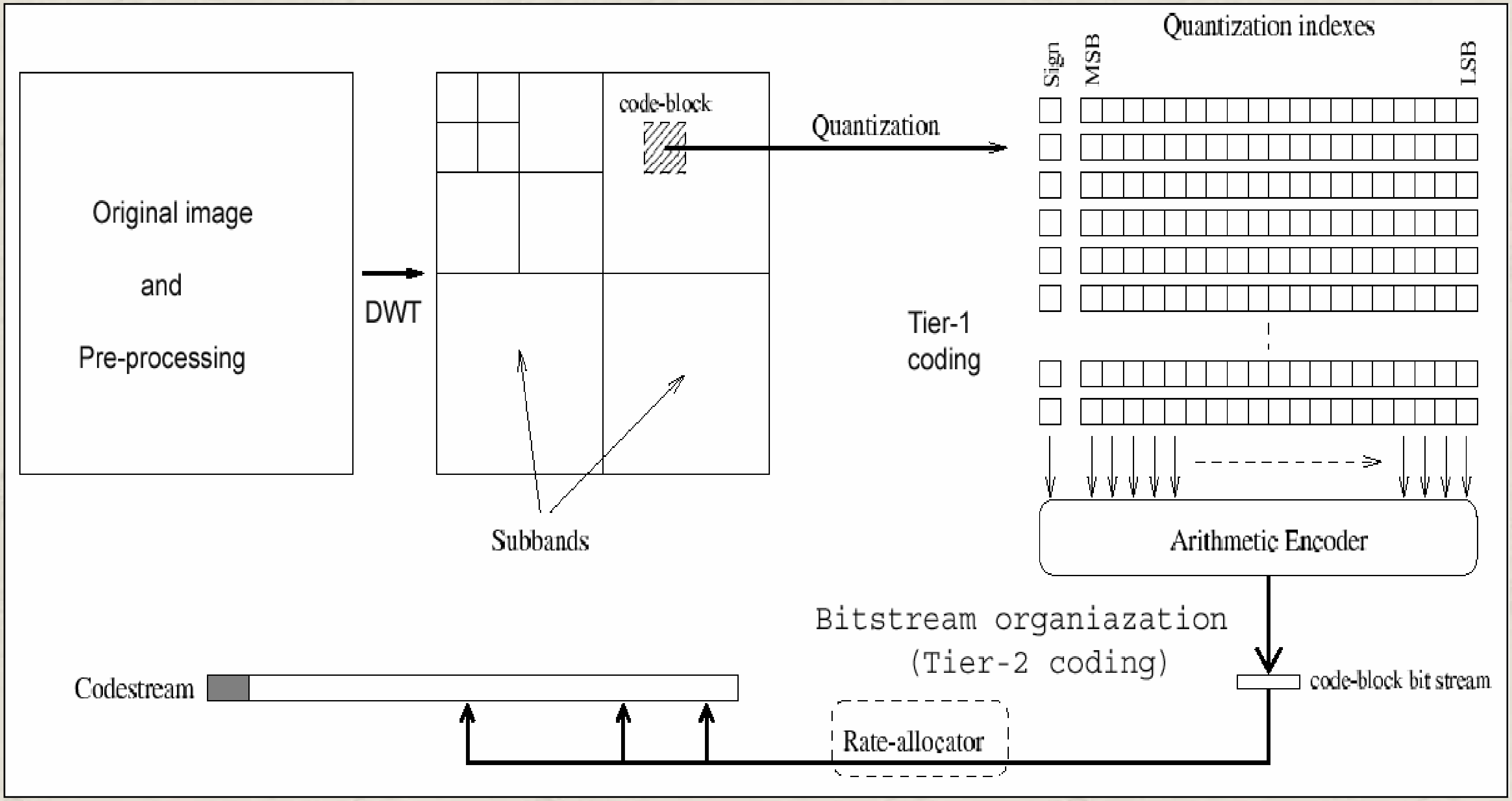



Decoder choices:

image resolution,
image fidelity,
region-of-interest,
fixed-rate,
components

JPEG 2000 offers flexible decoding

# JPEG2000 Compression Scheme



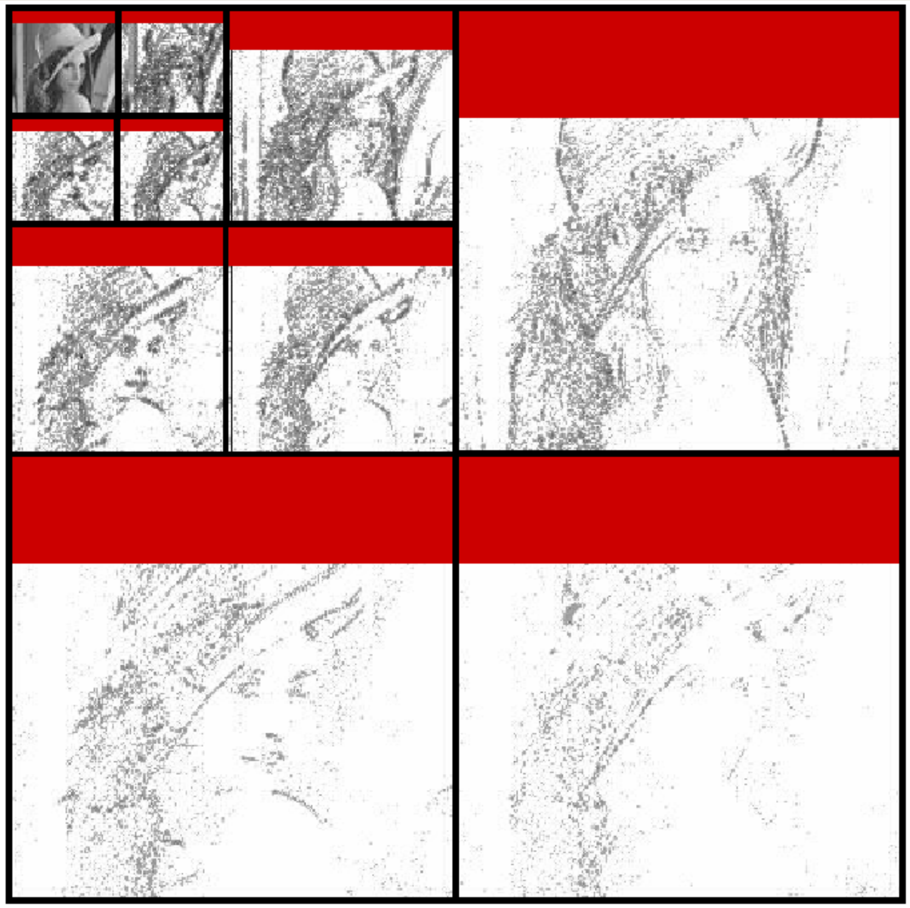

R. Grosbois, *et.al.*, “New approach to JPEG2000 compliant Region-of-Interest coding”, Proc. of the SPIE 46th Annual Meeting, San Diego, CA, 2001

# Part 1: Discrete Wavelet Transform

- Inherent to normal DWT:
  - Multi-resolution image representation
  - Eliminate blocking artifacts at high compression ratio
  - Each subband can be quantized differently
- Special techniques:
  - Provide integer filter (e.g. (5,3) filter) to support lossless and lossy compression within a single compressed bit-stream;
  - Line-based DWT and lifting implementations to reduce the memory requirement and computational complexity.

Except for a few special case, e.g., the (5,3) integer filter, the DWT is generally more computationally complexity (~2 to 3) than the block-based DCT; and DWT also requires more memory than DCT.

# Line-based DWT Implementation



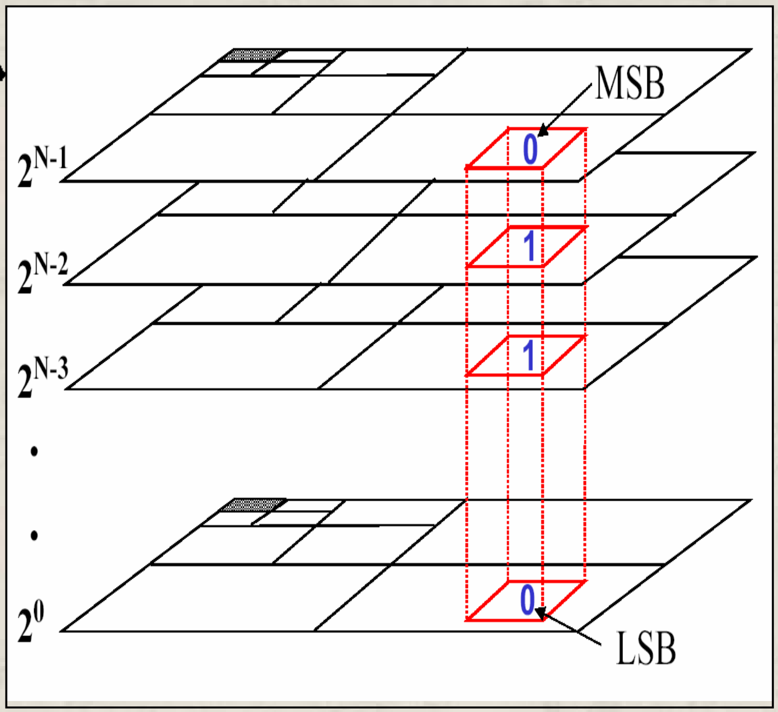
- There is no need to buffer an entire image in order to perform wavelet transform
- Depending on filter lengths and decomposition levels, a line of wavelet coefficients can be made available only after processing a few lines of the input image

# Part 2: Quantization

- Embedded Quantization

  Quantization index is encoded bit by bit, starting from Most Significant Bit (MSB) to Least Significant Bit (LSB)

- Example




Wavelet coefficient = 209

Quantizer step size $\Delta_b = 2$

Quantization index = $\lfloor 209/2 \rfloor = 104$
= 01101000;

Dequantized value based on fully decoded index:
(104+0.5)*2 = 209;

Decoding value after decoding 3 bit planes:

•Decoded index = 011 = 3;

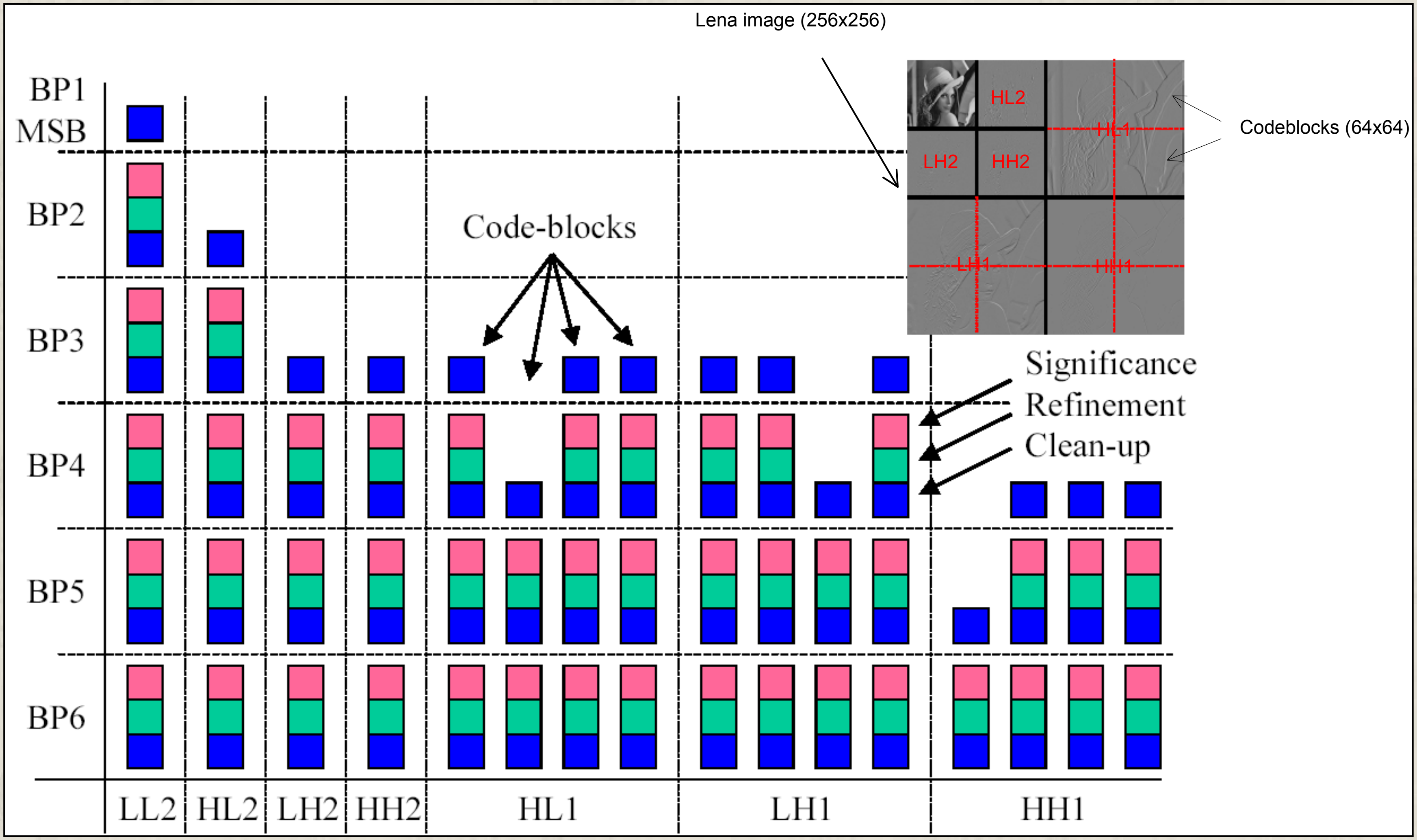
•Step size = 2*32=64

•Dequantized value = (3+0.5)*64 = 224

# Part 3: Entropy Coding (Tier-1 )

- Tier-1 Entropy coding
  - Each bit-plane is individually coded by the context-based adaptive binary arithmetic coding (JBIG2 MQ-coder)
  - Each bit plane is partitioned into blocks, named *code-blocks*, which are encoded independently
  - Each bit plane of each block is encoded in three *sub-bit-plane passes*
    - Significance propagation pass
    - Magnitude refinement pass
    - Clean-up pass

# Example of Bit-plane Coding

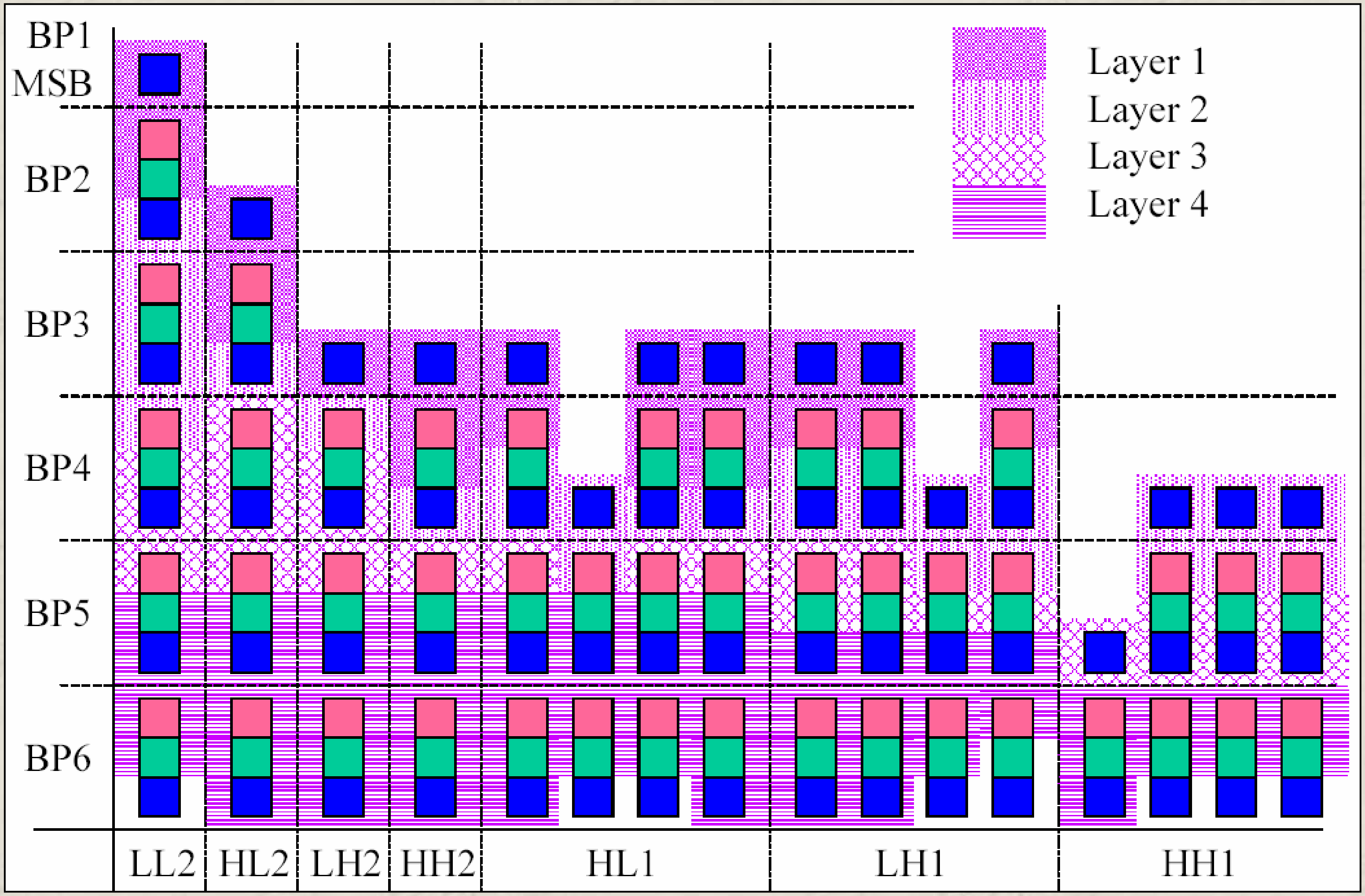



M. Rabbani, *et.al.*, “The JPEG2000 still image compression standard”, Proc. of ICIP, 2001

# Part 4: Bit stream Organization (Tier 2)

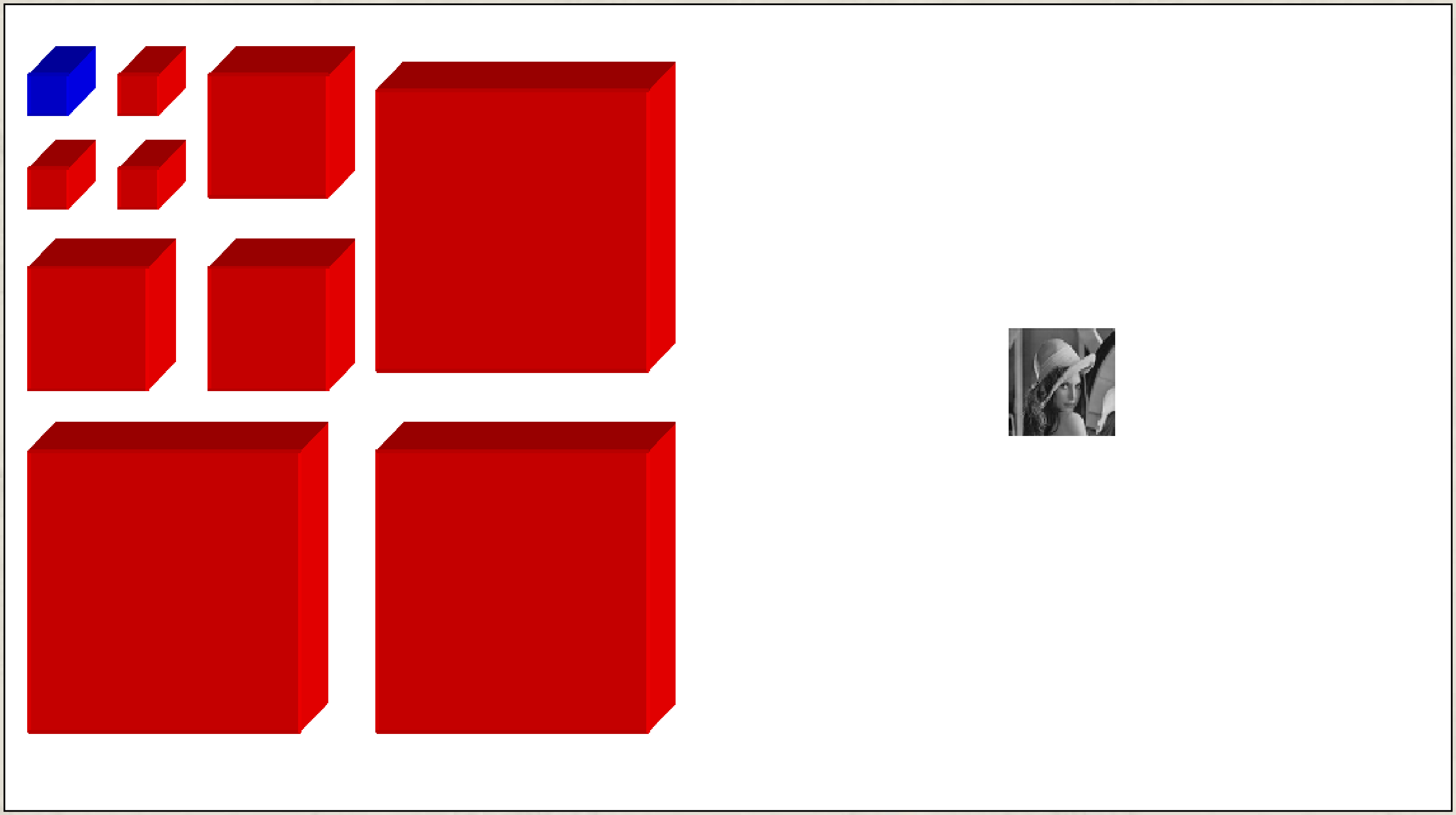
- Tier-1 generates a collection of bitstreams
  - One independent bitstream from each code block
  - Each bitstream is embedded
- Tier-2 multiplexes the bitstreams for inclusion in the codestream and signals the ordering of the resulting coded bitplane passes in an efficient manner.
- Tier-2 coded data can be rather easily parsed
- Tier-2 enables SNR, resolution, spatial, ROI and arbitrary progression and scalability

# Example: Bit-stream Organization




M. Rabbani, *et.al.*, “The JPEG2000 still image compression standard”, Proc. of ICIP, 2001

# Example: Progressive Resolution



# JPEG2000 Summary

- JPEG2000 offers the state-of-the-art features
  - Superior low bit rate performance and coding efficiency (up to 30% compared with DCT)
  - Lossless and lossy compression
  - Progressive transmission by pixel accuracy and resolution
  - Region-of-Interest coding
  - Random codestream access and processing
  - Error resilience
  - Open architecture
  - Content-based description
  - Side channel spatial information (transparency)
  - Protective image security
  - Continuous-tone and bi-level compression

# Summary

- Prediction
  - DPCM, generalized to linear prediction
- Transformation
  - Transform fundamentals
  - Karhunen-Loeve Transform (KLT): optimal linear transform
  - Discrete Cosine Transform (DCT): properties
  - Discrete Wavelet Transform (DWT): multi-resolution representation
- JPEG: first international compression standard for still images
  - DCT – Quantization – Run-length – Huffman
- JPEG2000: latest technology, wavelet-based
  - Scalable, progressive coding with flexible intelligent functionalities