Digital Signal Processing Summer Seminar
Institute of Information Technology - Vietnam Education Foundation
Image/Video Coding and Processing

Homework Assignment II

July 2006

I *Quantization.*
Suppose that a random variable X has the two-sided exponential PDF

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}.$$

Consider the design of a three-level midtread scalar quantizer Q as follows

$$Q(x) = \begin{cases} +b & x > a \\ 0 & -a \leq x \leq a \\ -b & x < -a \end{cases}$$

a. Sketch the probability density distribution of the given random variable with the case $\lambda = 1$.

b. Find an expression for b as a function of a so that the centroid condition is met.

c. For what value of a will the quantizer using b chosen as above satisfy *both* the Lloyd conditions for optimality?

d. What is the resulting mean squared error for your choices of a and b?

e. Specialize your answers of Part **b, c, d** to the case $\lambda = 1$.

f. Now design the optimal 1-bit quantizer for the same PDF. What are the decision boundary and the reconstructed values so that the quantizer is optimal in the mean-squared sense?

II *DCT factorizations.*
The 4 types of 1D M-point forward discrete cosine transform (DCT) are defined below (not in normalized form)

$$\text{Type-I:} \quad X_m = c_m \sum_{n=0}^{M} c_n \, x_n \, cos \left[mn \frac{\pi}{M} \right], \quad m = 0, 1, \ldots, M$$

$$\text{Type-II:} \quad X_m = c_m \sum_{n=0}^{M-1} x_n \, cos \left[m \left(n + \frac{1}{2} \right) \frac{\pi}{M} \right], \quad m = 0, 1, \ldots, M-1$$

$$\text{Type-III:} \quad X_m = \sum_{n=0}^{M-1} c_n \, x_n \, cos \left[n \left(m + \frac{1}{2} \right) \frac{\pi}{M} \right], \quad m = 0, 1, \ldots, M-1$$

$$\text{Type-IV:} \quad X_m = \sum_{n=0}^{M-1} x_n \, cos \left[\left(m + \frac{1}{2} \right) \left(n + \frac{1}{2} \right) \frac{\pi}{M} \right], \quad m = 0, 1, \ldots, M-1$$

where

$$c_m = \begin{cases} \frac{1}{\sqrt{2}} & m = 0 \text{ or } M \\ 1 & \text{otherwise.} \end{cases}$$

a. Consider the Type-II DCT when M is even. Show that the even-indexed coefficients X_{2m} and the odd-indexed coefficients X_{2m+1} can be computed as follows:

$$\begin{aligned} X_2m &= c_m \sum_{n=0}^{\frac{M}{2}-1} [x_n + x_{M-1-n}] \, cos \left[m \left(n + \frac{1}{2} \right) \frac{\pi}{M/2} \right], \quad m = 0, 1, \ldots, \frac{M}{2} - 1 \\ X_{2m+1} &= \sum_{n=0}^{\frac{M}{2}-1} [x_n - x_{M-1-n}] \, cos \left[\left(m + \frac{1}{2} \right) \left(n + \frac{1}{2} \right) \frac{\pi}{M/2} \right], \quad m = 0, 1, \ldots, \frac{M}{2} - 1. \end{aligned}$$

b. Show that any M-point DCT-II (M even) can be constructed from an $\frac{M}{2}$-point DCT-II and an $\frac{M}{2}$-point DCT-IV. Draw a general block diagram illustrating this property.

c. Prove that the DCT basis vectors are either symmetric or antisymmetric. Which basis vectors are symmetric? Which are antisymmetric?