DAY 2

Saturday, July 22rd, 2006

I. Lecture: (total 54 slides)

I.1. Modulation and Polyphase Representations: (25/27 slides)

1. Noble Identities
2. Block Toeplitz Matrices and Block z-transforms
3. Polyphase Examples

I.2. Orthogonal Wavelet Bases: (18/18 slides)

1. Connection to Orthogonal Filters
2. Orthogonality in the Frequency Domain
3. Biorthogonal Wavelet Bases

I.3. Maxflat Filters: (11/18 slides)

1. Daubechies and Meyer Formulas
2. Spectral Factorization

Link:

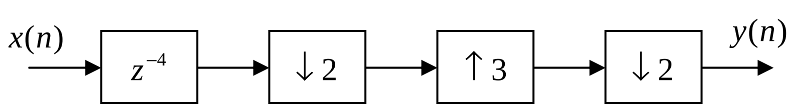
- http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/3019274A-2431-4C96-8FD6-BC96B330AA52/0/Slides5.pdf
- http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/16E2D430-C01C-49BF-981F-0ED675817545/0/Slides7.pdf
- http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/9B97D14A-5F49-4F2E-85B0-47B51339D941/0/Slides8.pdf

II. Exercise and Lab:

II.1. Exercise:

1. Problem Set 3.4:

   Problems 3: Simplify the following system:



What is $Y(z)$ in term of $X(z)$? Find $y(n)$ for the following inputs: $x(n)=\delta(n)$, $x(n)=(\ldots, \quad 1, \quad 1, \quad 1, \quad 1, \quad \ldots)$, $x(n)=(\ldots, \quad 1, \quad -1, \quad 1, \quad -1, \quad \ldots)$.

2. Problem Set 4.2:

<u>Problems 1</u>: Find $X_{even}(z)$ and $X_{odd}(z)$ when $X(z)=1+2z^{-5}+z^{-10}$. Verify that $X_{even}(z^2)=\frac{1}{2}\left(X(z)+X(-z)\right)$ and $X_{odd}(z^2)=\frac{z}{2}\left(X(z)-X(-z)\right)$. The odd definition involves an advance!.

<u>Problem 4</u>: Polyphase Representation of an IIR Transfer function

Let $H(z)=\dfrac{1}{1-az^{-1}}$ where $0<a<1$. Its impulse response is $h(n)=a^n$ for $n\geq 0$ (and zero for negative n). The phases are $h_{even}(n)=(1, \quad a^2, \quad a^4, \quad \ldots)$ and $h_{odd}(n)=(a, \quad a^3, \quad a^5, \quad \ldots)$. The z-transform are $H_{even}(n)=\dfrac{1}{(1-a^2z^{-1})}$ and $H_{odd}(n)=\dfrac{a}{(1-a^2z^{-1})}$. This method is very cumbersome. One has to find the impulse response $h(n)$, then its even and odd parts $h_{even}(n)$ and $h_{odd}(n)$, then the z-transform.

An alternative method is to write $H(z)=\dfrac{1}{1-az^{-1}}$ directly as $H(z)=H_{even}(z^2)+z^{-1}H_{odd}(z^2)$. The dominator must be a function of z^2. So multiply above and below by $1+az^{-1}$:

$$H(z)=\frac{1}{1-az^{-1}}\frac{1+az^{-1}}{1+az^{-1}}=\frac{1+az^{-1}}{1-a^2z^{-2}}=\frac{1}{1-a^2z^{-2}}+z^{-1}\frac{a}{1-a^2z^{-2}}$$

This displays $H_{even}(z)$ and $H_{odd}(z)$. An N^{th} order filter can be factored as a cascade of first-order sections, and this method applies to each section.

(a). Let $H(z)=\dfrac{1}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$. Factor $H(z)$ into two first-order poles. Find the polyphase components of $H(z)$.

(b). Let $H(z)=\dfrac{1+2z^{-1}+5z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$. What are this polyphase components?

<u>Problem 7</u>: Let $H(z)=1+2z^{-1}+3z^{-2}+4z^{-3}+4z^{-4}+3z^{-5}+2z^{-6}+z^{-7}$. Find the polyphase components $H_{even}(z)$ and $H_{odd}(z)$ for antisymmetric filters of even length and symmetric filters of odd length?

3. Problem Set 4.3:

<u>Problem 2</u>: If $H_m^T(z^{-1})H_m(z) = 2I$ show from

$$\begin{bmatrix} H_{0,even}(z^2) & H_{0,odd}(z^2) \\ H_{1,even}(z^2) & H_{1,odd}(z^2) \end{bmatrix} \begin{bmatrix} 1 & \\ & z^{-1} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} H_0(z) & H_0(z) \\ H_1(z) & H_1(z) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

that $H_p^T(z^{-1})H_p(z) = I$

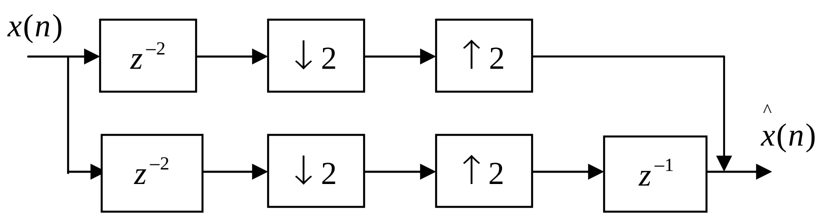
<u>Problem 17</u>: Find the analysis filters $H_0(z)$ and $H_1(z)$ for the following matrices:

(b) $H_p(z) = \begin{bmatrix} 1+2z^{-2} & 1+z^{-1} \\ 1-z^{-1} & 2+z^{-1} \end{bmatrix} \begin{bmatrix} 2 & 1+z^{-2} \\ 1-z^{-1} & -z^{-3} \end{bmatrix}$.

(c) $H_p(z) = \begin{bmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & s_2 \\ -s_2 & c_2 \end{bmatrix}$.

4. Problem Set 4.4:

<u>Problem 9</u>: Find the matrices $H_p(z)$ and $F_p(z)$. Is the system PR?



5. Problem Set 5.2:

<u>Problem 2</u>: For any four coefficients $h(0),...,h(3)$, verify that

$$|H_{even}(z)|^2 + |H_{odd}(z)|^2 = \frac{1}{2}\left(|H(z)|^2 + |H(-z)|^2\right)$$

Then condition O for the polyphase equals Condition O for modulation.

<u>Problem 4</u>: Find d by alternating flip of $c = (c(0),...,c(5))$. Verify equation $\sum c(n)d(n-2k) = 0$ directly to show that c is double-shift orthogonal to d.

6. Problem Set 5.4:

<u>Problem 3</u>: Why must all roots of $P(z)$ on the unit circle have even multiplicity, to allow $P(z) = C(z)C(z^{-1})$ and $P(\omega) = |C(\omega)|^2$?

II.2. Matlab:

1. Magnitude and Phase response of Daubechies 4 -tap filter: use the cepstral method to write a function $h_0 = \text{daub}(Nh)$ to find the Daubechies analysis filter $h_0(n)$ with length Nh:

$$P(z) = 2\left(\frac{1+z}{2}\right)^p \left(\frac{1+z^{-1}}{2}\right)^p \sum_{k=0}^{p-1} \binom{p+k-1}{k} \left(\frac{1-z}{2}\right)^k \left(\frac{1-z^{-1}}{2}\right)^k = 2\left(\frac{1+z}{2}\right)^p \left(\frac{1+z^{-1}}{2}\right)^p Q(z)$$

$$= \left(\frac{1+z}{2}\right)^p \left(\frac{1+z^{-1}}{2}\right)^p R(z)R(z^{-1}) = \left(\left(\frac{1+z}{2}\right)^p R(z)\right)\left(\left(\frac{1+z^{-1}}{2}\right)^p R(z^{-1})\right)$$

- Determine the M point DFT of $Q(k)$. Use the equation:

$$Q(z) = \sum_{k=0}^{p-1} \binom{p+k-1}{k} \left(\frac{1+z}{2}\right)^k \left(\frac{1-z}{2}\right)^k$$

  Where $p = Nh/2$, $z = e^{j2\pi k/N}$, $N = 512$ FFT point, $k = 0,1,...,N$

  Type:

```
K = Nh/2;
L = Nh/2;
N = 512;                               % Use a 512 point FFT by default.
k = 0:N-1;
z = exp(j*2*pi*k/N);
tmp1 = (1 + z.^(-1)) / 2;
tmp2 = (-z + 2 - z.^(-1)) / 4;         % sin^2(w/2)
Mz1 = zeros(1,N);
vec = ones(1,N);
for l = 0:K-1
  Mz1 = Mz1 + vec;
  vec = vec .* tmp2 * (L + l) / (l + 1);
end
Mz1 = 2 * Mz1;
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- Find the specstral of $Q(k)$ by using equation $\hat{q}(n) = iDFT(\ln(Q(k)))$.

- Find the causal part of $\hat{q}(n)$.

$$\hat{r}(n) = \begin{cases} 1/2\,\hat{q}(0) & \text{if } n = 0 \\ \hat{q}(n) & \text{if } n > 0 \\ 0 & \text{if } n < 0 \end{cases}$$

- Determine the DFT of $r(n)$ by equation $R(k) = e^{DFTt(\hat{r}(n))}$.
- Including half the zeros at $z = -1$ to get $H_0(k) = R(k)\left(1 + e^{-j\frac{2\pi k}{M}}\right)^p$ where $p = Nh/2$.
- Find $h_0(n) = iDFT(H_0(z))$.
- Find and plot the zero and pole of $P(k)$.

2. Develop a Matlab program to compute the spectral factors of a symmetric, positive definite filter: write function specfact.m with syntax [H0, H1] = specfact(p), where p is the order of the filter to compute the low and highpass orthogonal filters with p zeros at π by computing the roots of the product filter of degree $4p-2$.

- Use prodfilt.m to compute $P_0(z)$, $B(z)$ and $Q(z)$.
- Compute the roots r of $Q(z)$ and the roots within the unit circle r_0.
- Compute polynomial with roots r_0.
- Find binomial term with p zeros at π.
- Compute and normalize the lowpass filter and compute the highpass filter.

Link: http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/A318B5E6-442F-4EE0-B4B1-FD624C05B8ED/0/pset2.pdf