DAY 1

Friday, July 21st, 2006

I. Lecture: (total 55 slides)

I.1. Sampling Rate Change Operations: (12/20 slides)

1. Upsampling and Downsampling
2. Fractional Sampling
3. Interpolation

I.2. Filter Banks: (21/21+22/31=43/52 slides)

1. Time Domain (Haar example) and Frequency Domain
2. Conditions for Alias Cancellation and no Distortion
3. Perfect Reconstruction
4. Halfband Filters
5. Possible Factorizations

Link:

- http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/23AF6A07-9961-45A0-84CB-FC2223530785/0/Slides2.pdf
- http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/128CAA79-1201-42F7-AEF9-DB38F4974D12/0/Slides3.pdf
- http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/E3531682-5CFA-4FDB-8D4F-4A31A7A79893/0/Slides4.pdf

II. Exercise and Lab:

II.1. Exercise:

1. Problem Set 1.3:

Problems 1: Can a symmetric filter, with $h(k) = h(N-k)$, be a highpass filter?

Problems 3: Which of the following filters are invertible? Find the inverse filters:

a) $h(0) = \frac{2}{3}$ and $h(1) = -\frac{1}{3}$

b) $h(0) = 2$ and $h(2) = 1$

c) $h(n) = \frac{1}{n!}$ ($n = 0,1,2,\ldots$)

2. Problem Set 1.4:

   Problems 9: In a transmultiplexer, the synthesis bank comes before the analysis bank. Compute LL^T and LB^T and BB^T to verify that the Haar transmultiplexer still give perfect construction:

$$\begin{bmatrix} L \\ B \end{bmatrix}\begin{bmatrix} L^T & B^T \end{bmatrix} = \begin{bmatrix} LL^T & LB^T \\ BL^T & BB^T \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

   Problems 11: If $H_0(z) = 1$ and $H_1(z) = z^{-1}$ (no filtering) write the entries of $\begin{bmatrix} L \\ B \end{bmatrix}\begin{bmatrix} L^T & B^T \end{bmatrix} = I$.

3. Problem Set 3.3:

   Problems 2: Verify the theorem 3.6 directly:

   Theorem 3.6: the operator $(\downarrow L)$ and $(\uparrow M)$ commute if and only if L and M are relative prime. Their greatest common divisor is $(L, M) = 1$. In that case $(\uparrow L)(\downarrow M)x = (\downarrow M)(\uparrow L)x$ has components

$$u(n) = \begin{cases} x(Mk) & \text{if } n/L = k \text{ is an integer} \\ 0 & \text{if } n/L \text{ is not an integer} \end{cases}$$

   What happens to the odd-numbered components of x when we compute $(\uparrow 2)(\downarrow 2)x$?

   Problems 7: In smoothing $u(n)$ to get the final output $w = Fu$, which filters F will interpolate and not change the even samples: $w(2k) = u(2k)$?

4. Problem Set 4.1:

   Problem 3: Find all filters if $H_0(z) = \left(\frac{1+z^{-1}}{2}\right)^3$ and $P_0(z) = \frac{1}{16}\left(-1 + 9z^{-2} + 16z^{-3} + 9z^{-4} - z^{-6}\right)$.

   Problems 4: If an FIR filter $H_0(z)$ has three or more coefficients, explain why $H_0^2(z)$ has at least two odd powers. Then $H_0^2(z) - H_0^2(-z) = 2z^{-l}$ is impossible. The "alternating signs" construction is not PR.

   Problems 9: the 10th degree halfband polynomial $P_0(z) = \left(1 + z^{-1}\right)^6 Q(z)$ has four complex roots r, $\bar{r}$, r^{-1}, $\bar{r}^{-1}$ in the right halfplane (roots of Q). Draw a figure to show the ten roots and how Daubechies 6/6 filters will divide them: r and $\bar{r}$ are separated from r^{-1} and $\bar{r}^{-1}$.

Problems 11: Find the actual 4^{th} degree $Q(z)$ that makes $P_0(z)$ halfband. If possible compute its roots.

II.2. Matlab:

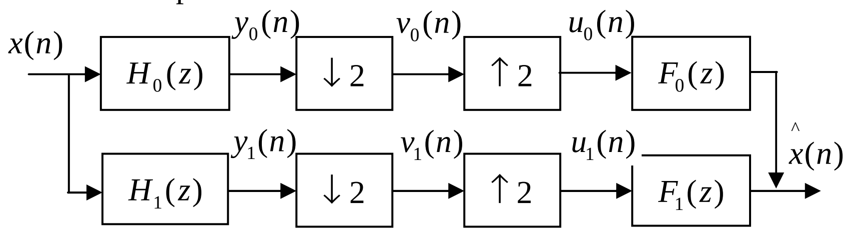
1. Haar highpass and lowpass filter:
   - Plot the frequency response ($N = 1024$ point) of Haar lowpass filter: $h = \left[\frac{1}{2}, \quad \frac{1}{2}\right]$.

     Type:

     ```
     N = 1024;
     W = (-N/2:N/2-1) / (N/2);
     % Low pass filter.
     h0 = [0.5, 0.5];
     H0 = fftshift(fft(h0,N));
     plot(W,abs(H0))
     ```

   - Plot the frequency response ($N = 1024$ point) of Haar highpass filter: $h = \left[\frac{1}{2}, \quad -\frac{1}{2}\right]$.
   - Plot the frequency response ($N = 1024$ point) of linear interpolating lowpass filter: $h = \left[\frac{1}{2}, \quad 1, \quad \frac{1}{2}\right]$.
2. Upsampling, downsampling:
   - Plot the Fourier transform ($N = 1024$ point) of $\left[\frac{1}{2}, \quad 0, \quad \frac{1}{2}, \quad 0\right]$.
   - Plot the Fourier transform ($N = 1024$ point) of $x = \left[\frac{-1}{16}, \quad 0, \quad \frac{9}{16}, \quad 1, \quad \frac{9}{16}, \quad 0, \quad \frac{-1}{16}\right]$.
   - Plot the Fourier transform ($N = 1024$ point) of $x_2 = \left[\frac{-1}{16}, \quad \frac{9}{16}, \quad \frac{9}{16}, \quad \frac{-1}{16}\right]$.
   - Plot $\frac{1}{2}\left(X\left(\frac{\omega}{2}\right) + X\left(\frac{\omega}{2} + \pi\right)\right)$ and compare to $X_2(\omega)$.

3. Analysis two channel perfect reconstruction filter bank:



➢ Consider a two channel perfect reconstruction filterbank with the analysis filters $h_0 = \left[\frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}}\right]$ and $h_1 = \left[\frac{1}{\sqrt{2}}, \quad -\frac{1}{\sqrt{2}}\right]$. Consider also a signal $x(n) = [0, \quad 1, \quad -1, \quad 2, \quad 5, \quad 1, \quad 7, \quad 0]$.

a) Find the corresponding dual (synthesis) filters $f_0(n)$, $f_1(n)$. Hint: use the condition for alias cancellation: $F_0(z) = H_1(-z)$ and $F_1(z) = -H_0(-z)$.

b) Find (and plot) the signals $v_0(n)$, $v_1(n)$ and $x(n)$.

c) Plot the zeros of the filters $h_0(n)$, $h_1(n)$, $f_0(n)$ and $f_1(n)$ (use zplane).

d) Plot the frequency spectra of $v_0(n)$ and $v_1(n)$.

e) Find (and plot) the signals $u_0(n)$, $u_1(n)$ and $\hat{x}(n)$.

f) Plot the frequency spectra of $u_0(n)$ and $u_1(n)$

➢ Repeat the previous problem for the filters $h_0(n) = \left[\frac{-1}{4\sqrt{2}}, \quad \frac{2}{4\sqrt{2}}, \quad \frac{6}{4\sqrt{2}}, \quad \frac{2}{4\sqrt{2}}, \quad \frac{-1}{4\sqrt{2}}\right]$ and $h_1(n) = \left[\frac{1}{2\sqrt{2}}, \quad \frac{-2}{2\sqrt{2}}, \quad \frac{1}{2\sqrt{2}}\right]$.

Link: http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/BFE0EB1B-06F7-43F5-AED2-9AE336F4AF88/0/pset1.pdf

4. Product filter : use the build-in function prodfilt.m to find the halfband product filter of degree $4p-2$ then plot the zeros and frequency response of that halfband product filter.

5. Practical Problem:

Description: The Frequency Response specification attempts to describe the range of frequencies or musical tones a speaker can reproduce, measured in Hertz (Cycles per Second). The range of human hearing is generally regarded as being from 20Hz, very low bass tones, through 20kHz (20,000Hz), the very highest treble. A typical 3-way loudspeaker has three types of speaker: a Subwoofer, a Midrange and a Treble.

Your task: Design a speaker with the following requirements:

1. A **Subwoofer** is a speaker dedicated to the reproduction of low frequencies, design a filter to produce a subwoofer with the frequency response from about 20 Hz to about 200 Hz.
2. A **Midrange** is a type of speaker dedicated to the reproduction of midrange frequencies, design a filter to produce a subwoofer with the frequency response from about 400 Hz to about 1kHz.
3. A **Treble** is a type of speaker dedicated to the reproduction of high frequencies, design a filter to produce the a subwoofer with the frequency response from about 1k Hz to about 20kHz

Note: Show frequency response of each speaker. Then apply the algorithm to an audio file and play the subband signals for validation.