## Day 1

1. Problem Set 1.3:

<u>Problems 1</u>: Can a symmetric filter, with h(k) = h(N - k), be a highpass filter?

Problems 3: Which of the following filters are invertible? Find the inverse filters:

a) 
$$h(0) = \frac{2}{3}$$
 and  $h(1) = -\frac{1}{3}$   
b)  $h(0) = 2$  and  $h(2) = 1$   
c)  $h(n) = \frac{1}{n!}$   $(n = 0, 1, 2, ...)$ 

2. Problem Set 1.4:

<u>Problems 9</u>: In a transmultiplexer, the synthesis bank comes before the analysis bank. Compute  $LL^{T}$  and  $LB^{T}$  and  $BB^{T}$  to verify that the Haar transmultiplexer still give perfect construction:

$$\begin{bmatrix} L \\ B \end{bmatrix} \begin{bmatrix} L^T & B^T \end{bmatrix} = \begin{bmatrix} LL^T & LB^T \\ BL^T & BB^T \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

<u>Problems 11</u>: If  $H_0(z) = 1$  and  $H_1(z) = z^{-1}$  (no filtering) write the entries of  $\begin{bmatrix} L \\ B \end{bmatrix} \begin{bmatrix} L^T & B^T \end{bmatrix} = I$ .

3. Problem Set 3.3:

Problems 2: Verify the theorem 3.6 directly:

**Theorem 3.6**: the operator  $(\downarrow L)$  and  $(\uparrow M)$  commute if and only if L and M are relative prime. Their greatest common divisor is (L, M) = 1. In that case  $(\uparrow L)(\downarrow M)x = (\downarrow M)(\uparrow L)x$  has components

 $u(n) = \begin{cases} x(Mk) & \text{if } n/L = k \text{ is an integer} \\ 0 & \text{if } n/L \text{ is not an integer} \end{cases}$ 

What happens to the odd-numbered components of x when we compute  $(\uparrow 2)(\downarrow 2)x$ ?

**2.** Verifying (3.27):  $\mathbf{v} = (\downarrow M)\mathbf{x}$  and  $\mathbf{u} = (\uparrow L)\mathbf{v}$  have components:

$$\mathbf{v}(k) = \mathbf{x}(Mk)$$
$$\mathbf{u}(n) = \begin{cases} \mathbf{v}(k) = \mathbf{x}(Mk) = \mathbf{x}(\frac{Mn}{L}), & \text{if } \frac{n}{L} = k \text{ is integer}\\ 0, & \text{otherwise} \end{cases}$$

Change the order of  $(\downarrow M)$  and  $(\uparrow L)$ : upsampling first puts in L-1 zeros between each  $\mathbf{x}(n)$  and  $\mathbf{x}(n+1)$ . It has components:

$$\mathbf{v}'(n) = \begin{cases} \mathbf{x}(k) = \mathbf{x}(\frac{n}{L}), & \text{if } \frac{n}{L} = k \text{ is an integer} \\ 0, & \text{otherwise} \end{cases}$$

Downsampling  $\mathbf{u}' = (\downarrow M)\mathbf{v}'$  keeps every *M*th components of  $\mathbf{v}'$  and removes all the other components:

$$\mathbf{u}'(n) = \mathbf{v}'(Mn) = \begin{cases} \mathbf{x}(k) = \mathbf{x}(\frac{Mn}{L}), & \text{if } \frac{Mn}{L} = k \text{ is an integer} \\ 0, & \text{otherwise} \end{cases}$$

$$(M,L) = 1 \iff \frac{Mn}{L}$$
 is integer *if and only if*  $\frac{n}{L}$  is integer  
 $\iff \mathbf{u}'(n) = \mathbf{u}(n)$ 

Therefore,  $(\uparrow L)(\downarrow M)\mathbf{x} = (\downarrow M)(\uparrow L)\mathbf{x}$  if and only if L and M are relatively prime. The odd-numbered components become zeros after  $(\uparrow 2)(\downarrow 2)$ .

In  $X(z) = \sum \mathbf{x}(n)z^{-n}$ , the odd-numbered coefficients are zero, therefore  $X(z) = \sum \mathbf{x}(2n)z^{-2n}$ .

<u>Problems 7</u>: In smoothing u(n) to get the final output w = Fu, which filters F will interpolate and not change the even samples: w(2k) = u(2k)?

7.  $\mathbf{f} = (...0, \frac{1}{2}, 1, \frac{1}{2}, 0...)$ . This filter keeps the even samples and replaces the zero values in the odd-numbered components  $\mathbf{u}(2k+1)$  with a simple "average":  $\frac{\mathbf{u}(2k)+\mathbf{u}(2k+2)}{2}$ . The matrix form is:

$\frac{(-4) + \mathbf{u}(-2)}{\mathbf{u}(-2)}$ $\frac{\mathbf{u}(-2) + \mathbf{u}(0)}{\mathbf{u}(0) + \mathbf{u}(2)}$ $\frac{\mathbf{u}(0) + \mathbf{u}(2)}{\mathbf{u}(2)}$ $\frac{\mathbf{u}(2) + \mathbf{u}(4)}{2}$ $\vdots$	=	$\begin{array}{c} \cdot \\ 0 \\ \mathbf{u}(-2) \\ 0 \\ \mathbf{u}(0) \\ 0 \\ \mathbf{u}(2) \\ 0 \\ \cdot \\ \cdot \end{array}$		$     \begin{array}{c}             0 \\             0 \\         $	$\begin{array}{c} \cdot \\ 0 \\ 0 \\ \frac{1}{2} \\ 1 \\ \frac{1}{2} \\ \cdot \end{array}$	
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In general, a halfband filter satisfying  $\sum_{n} h(n)$  will interpolate and not change the even samples.

## 4. Problem Set 4.1:

Problem 3: Find all filters if 
$$H_0(z) = \left(\frac{1+z^{-1}}{2}\right)^3$$
 and  $P_0(z) = \frac{1}{16}\left(-1+9z^{-2}+16z^{-3}+9z^{-4}-z^{-6}\right)$ 

**3.**  $P_0(z)$  has six roots. Four real roots correspond to the four zeros of  $(1+z^{-1})^4$  at z = -1. The other two complex roots are  $c = 2 - \sqrt{3}$ ,  $\frac{1}{c} = 2 + \sqrt{3}$ . Given one factor  $H_0(z) = (\frac{1+z^{-1}}{2})^3$ , roots  $c, \frac{1}{c}$  and -1 will go into the synthesis filter  $F_0(z)$ . This factorization yields:

$$H_0(z) = \left(\frac{1+z^{-1}}{2}\right)^3 = \frac{1}{8}\left(1+3z^{-1}+3z^{-2}+z^{-3}\right)$$
  

$$F_0(z) = \left(\frac{1+z^{-1}}{2}\right)\left(c-z^{-1}\right)\left(\frac{1}{c}-z^{-1}\right) = \frac{1}{2}\left(-1+3z^{-1}+3z^{-2}-z^{-3}\right)$$

For aliasing cancellation, we choose

$$H_1(z) = F_0(-z)$$
 and  $F_1(z) = -H_0(-z)$ 

Therefore, the highpass filters are

$$H_1(z) = \frac{1}{2}(-1 - 3z^{-1} + 3z^{-2} + z^{-3})$$
  

$$F_1(z) = \frac{1}{8}(-1 + 3z^{-1} - 3z^{-2} + z^{-3})$$

<u>Problems 4</u>: If an FIR filter  $H_0(z)$  has three or more coefficients, explain why  $H_0^2(z)$  has at least two odd powers. Then  $H_0^2(z) - H_0^2(-z) = 2z^{-l}$  is impossible. The "alternating signs" construction is not PR.

4. See the proof of Theorem 5.4 on page 159.

<u>Problems 9</u>: the 10<sup>th</sup> degree halfband polynomial  $P_0(z) = (1 + z^{-1})^6 Q(z)$  has four complex roots r,  $\bar{r}$ ,  $r^{-1}$ ,  $\bar{r}^{-1}$  in the right halfplane (roots of Q). Draw a figure to show the ten roots and how Daubechies 6/6 filters will divide them: r and  $\bar{r}$  are separated from  $r^{-1}$  and  $\bar{r}^{-1}$ .

**9.**  $\mathbf{Q}(z) = \frac{1}{256}(3 - 18z^{-1} + 38z^{-2} - 18z^{-3} + 3z^{-4})$  is chosen so that  $P_0(z) = (1 + z^{-1})^6 \mathbf{Q}(z)$  has only one odd term  $z^{-5}$  (with coefficient 1). Then the 10th degree halfband polynomial is:

$$P_0(z) = \frac{1}{256} (3 - 25z^{-2} + 150z^{-4} + 256z^{-5} + 150z^{-6} - 25z^{-8} + 3z^{-10})$$

 $(1+z^{-1})^6$  has six zeros at z = -1. To find the roots of  $\mathbf{Q}(z)$ , we multiply  $\mathbf{Q}(z)$  by  $z^2$ . Then we get a quadratic formula for  $z + \frac{1}{z}$ :

$$3(z + \frac{1}{z})^2 - 18(z + \frac{1}{z}) + 32 = 0$$
$$z + \frac{1}{z} = 3 + \frac{\sqrt{15}}{3}i \quad \text{and} \quad z + \frac{1}{z} = 3 - \frac{\sqrt{15}}{3}i$$

This gives another four complex roots of  $P_0(z)$  (compare the solution to problem 2 in §5.5):

$$r = \frac{3 - \frac{\sqrt{15}}{3}i - \sqrt{\frac{10}{3} - 2\sqrt{15}i}}{2} \qquad r^{-1} = \frac{3 - \frac{\sqrt{15}}{3}i + \sqrt{\frac{10}{3} - 2\sqrt{15}i}}{2}$$
$$\bar{r} = \frac{3 + \frac{\sqrt{15}}{3}i - \sqrt{\frac{10}{3} + 2\sqrt{15}i}}{2} \qquad \bar{r}^{-1} = \frac{3 + \frac{\sqrt{15}}{3}i + \sqrt{\frac{10}{3} + 2\sqrt{15}i}}{2}$$

r and  $\bar{r}$  are inside the unit circle. Their reciprocals are outside. We plot the ten roots below. Daubechies 6/6 filters put  $r - z^{-1}$ ,  $\bar{r} - z^{-1}$ ,  $(1 + z^{-1})^3$  (the roots inside the unit circle) into  $H_0(z)$  and  $\frac{1}{r} - z^{-1}$ ,  $\frac{1}{\bar{r}} - z^{-1}$ ,  $(1 + z^{-1})^3$  into  $F_0(z)$  so that the analysis filter has minimum phase. The splitting is also shown in the figure. This construction is orthogonal.



• = roots of  $H_{\theta}$  and  $\times$  = roots of  $F_{\theta}$